Fine, Kit, *The Limits of Abstraction*, Oxford: Clarendon Press, 2002, pp. x+203, £18.99 (cloth).

The Limits of Abstraction slightly expands a long article previously published under the same name.[1] It is a great book. In demanding but always succinct prose, it presents a wealth of distinctions, insights, and arguments; its mathematical sophistication is impressive; and, perhaps because its author is motived more from "a sense of curiosity than commitment" [ix], it provides a refreshing approach to its topic: abstraction principles.

An *abstraction principle* is a principle of the form

(*) $\S\alpha = \S\beta \leftrightarrow \alpha \sim \beta$

where α and β are variables, and where $\sim$ is an equivalence relation. The most exciting such principles, which are the chief concern of the book, are *conceptual* abstraction principles, where α and β are second-order variables (which Fine takes to range over concepts). Beginning with Frege, many philosophers and logicians have thought that such principles can provide a grounding of classical (and thus platonistic) mathematics in pure logic (perhaps supplemented with other considerations that are no more problematic philosophically). Frege's own logical system consisted of second-order logic and the abstraction principle whose relation $\sim$ is coextensionality. But as Russell's paradox demonstrates, this system is inconsistent. In recent decades, however, *neo-logicists* such as Crispin Wright and his associates have shown that there are other abstraction principles which are both consistent and rich with mathematical consequences. The most famous example is Hume's Principle (HP), where the relation $\sim$ is equinumerosity (which can be expressed in pure second-order logic). The second-order theory

[1] In *Philosophy of Mathematics Today*, Matthias Schirn, ed. (Oxford: Clarendon Press, 1998).

with HP as its sole non-logical axiom is consistent and suffices, along with some natural definitions, to derive all of second-order Peano Arithmetic.

Despite this promising beginning, neo-logicism faces two serious problems, both having to do with the "goodness" of abstraction principles as a basis for mathematics. First, wherein does the *philosophical* goodness of good abstraction principles consist? What makes such principles suited to answering the difficult semantic and epistemic questions raised by classical mathematics? Second, can we give an informative general characterization of the abstraction principles that are *mathematically* good? One reason it is important to do so is the *Bad Company Problem*, which is that mathematically good abstraction principle, such as HP, are accompanied by mathematically bad ones, such as Frege's inconsistent principle.

Fine's book makes extremely valuable contributions to the discussion of both of these problems. The first two of the book's four parts, which deal primarily with the philosophical problem, deserve to be widely read. The final two parts, which attempt to solve the mathematical problem by developing a general theory of abstraction, are technically quite demanding and likely to be read in full only by people especially interested in the topic. However, working through all four parts is highly recommended, especially as the philosophical and the technical issues are so tightly interwoven. All that is presupposed is knowledge of basic logic and set theory—plus a good deal of determination.

Fine discusses three kinds of philosophical accounts of what a good abstraction principle accomplishes. According to the first kind of account, a good abstraction principle works as an *implicit definition*: it characterizes an operator mapping concepts to objects by laying down a condition that this operator must satisfy. This introduces two problems. What we may call the *Problem of Existence* is to show that there exists an operator satisfying this condition, and the *Problem of Uniqueness*, to show that this operator is unique (up to some desired equivalence). Fine argues, as against the neo-logicists, that the definition-mongerer is responsible for answering the Problem of Existence. He doubts that this kind of account contributes much to this task.

However, Fine thinks progress can be made on the Problem of Uniqueness by arguing that the abstraction principle gives *the essence* of the operator in question.

The second kind of account is that a good abstraction principle works by its left-hand side's being a *recarving* of the sense of its right-hand side. Although this view has received much attention from the neo-logicists, it is severely criticized in this book. Fine argues that the needed notion of recarving will overgenerate, and that it would anyway be at least as hard to establish the existence of a sense shared by the two sides as to solve the Problem of Existence directly.

The third and most ambitious kind of account regards a good abstraction principle as "creative": as *establishing*, rather than *presupposing*, the existence of the objects needed for the principle to be true. This is the theme of Part II, which is the book's major addition to the earlier article, and which is perhaps the best discussion of the topic in the literature. Inspired by Frege's Context Principle, Fine suggests we may "know that objects of the required sort exist by knowing that the definition has been referentially effective" [56]. This contrasts with the ordinary way of proceeding, which is the other way round. Roughly speaking, a definition of a term is "referentially effective" if it provides each sentence in which the term occurs with a suitable truth-condition, and if the truth-condition associated with some sentence whose truth requires the term to refer is in fact satisfied. Fine makes many interesting points about how this approach can be carried out and the extent to which it can deal with various aspects of the Problem of Uniqueness.

Particularly interesting is his discussion of a problem posed by a certain kind of impredicativity. For the approach outlined above to work, the truth-conditions assigned to sentences containing the terms to be defined must obviously be capable of being understood without any prior understanding of these terms. Does this mean that the truth-conditions in question cannot quantify over such entities as these terms purport to denote? Fine argues forcefully that it does, pointing out that since the purpose of this sort of definition "is to introduce a certain ontology of objects, we should not appeal to that ontology in explaining how the objects are to be introduced" [87]. If he is right, then the only abstraction principles capable of sustaining this approach will be *predicative* ones, that is, ones whose equivalence relation ~ does not quantify over such entities as the §-terms purport to denote. This is a severe restriction; in particular, it excludes the neo-logicists' favorite abstraction principle, HP.

In the end, Fine isn't entirely satisfied with any of the three kinds of philosophical accounts of abstraction principles that he discusses. But he reveals that his own sympathies lie with a version of the third kind of account—a version he calls "procedural postulationism" and promises to spell out in future work [v, 100].

I turn now to technical issues. The book opens by observing that, for a conceptual abstraction principle (*) to be true in a model, there must be a one-to-one function from the ~-equivalence classes of concepts to objects of the domain. Hence there can be no more ~-equivalence classes than there are objects in the domain. An equivalence relation ~ which satisfies this condition is called *non-inflationary*. If the concepts on a domain are represented by the full powerset of this domain (as Fine mostly assumes), then on a domain of cardinality κ there will be 2^κ concepts. This means that the non-inflationary equivalence relations will have to be quite coarse. For instance, equinumerosity is coarse enough, whereas coextensionality is much too fine.

This opening observation is then analyzed and generalized in various ways. Especially important to Fine are *systems* of abstraction principles. He plausibly assumes that two abstracts cannot be identical unless the associated equivalence classes of concepts are identical. But as he points out, there are more such equivalence classes of concepts than there are objects. This introduces a problem which Fine calls *hyper-inflation*. He proposes to deal with this problem by requiring that each equivalence relation ~ be *logical*, which he formally defines in terms of the invariance of ~ under permutations of the domain of objects. He then proves that systems of abstraction principles whose equivalence relations are logical and non-inflationary are immune to hyper-inflation, provided that the cardinality of the object domain has a certain property he calls *unsurpassability* [Theorem III.8.3].

In Part IV Fine develops a general theory of abstraction according to which all logical and non-inflationary equivalence relations can be (simultaneously) abstracted upon. This general theory is consistent—essentially because the Continuum Hypothesis, which is consistent, implies the existence of unsurpassable cardinals. Fine goes on to show how third-order Peano arithmetic can be interpreted in this (second-order) theory [IV.3]. This result is interesting because all Fine needs to assume is that abstraction principles with at most two equivalence classes are non-inflationary (because there are at least two objects), whereas the neo-logicists need to assume that HP is non-inflationary (and thus that there exist infinitely many objects). Conversely, John P. Burgess has shown that Fine's general theory of abstraction can be interpreted in third-order arithmetic.[2] Jointly, these two results draw "the limits of abstraction"— at least as analyzed by Fine.

Fine's book contains many other nice technical results. Many have to do with the generation of models of abstraction principles from below and the categoricity of the ensuing models. But this reviewer was particularly impressed by Theorem III.7.6, which shows that among the logical and non-inflationary equivalence relations on the concepts on an infinite domain there is a unique maximally fine one, which can be explicitly characterized in terms of coextensionality (on concepts with a certain smallness property and on their complements) and cardinality of the concept and of its complement (elsewhere).

Øystein Linnebo *University of Oslo*

[2] See his *Fixing Frege*, to appear with Princeton University Press.