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To Be Is to Be an *F*

1. Introduction

Is the natural number 3 identical with the Roman emperor Julius Caesar? In *Grundlagen* Frege raised some peculiar questions of this sort.[1] There are two kinds of intuitions regarding such questions. On the one hand, these questions seem not only to be pointless but to be downright meaningless. Regardless of how much arithmetic one studies, no answer to the opening question will be forthcoming. Arithmetic tells us that 3 is the successor of 2 and that it is prime, but not whether it is identical with Caesar. So questions concerning the identity of numbers and people seem not to be provided for. On the other hand, it seems that the opening question must be answered negatively. For numbers are not the sorts of things that *can* be identical with people. In fact, since the number 3 is an abstract object and Caesar is not, it follows by Leibniz's law that they are not identical.

The goal of this paper is to develop an account of reference and objecthood that does justice to both kinds of intuitions regarding cross-category identity statements. This is not an easy task. Many views, especially of a robust realist kind, fail to do justice to the first kind of intuitions. They regard it as completely obvious that, if *a* and *b* are genuine objects, the question whether *a* is identical with *b* must be meaningful and have a determinate answer (although not necessarily one we are able to discover).[2] Other views are unable to do justice to the second kind of intuitions. For instance, according to many structuralist views, the natural number 3 has no properties other than being the successor of 2, being prime, and other structural properties. On such views, 3 cannot have properties such as being non-identical with Caesar, since this would be a non-structural property.[3]

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[1] Actually, Frege never raises this particular question. However, since the first such question he discusses involves Caesar (Frege 1884, §56), this Roman emperor has become the stock example and has lent his name to the problem.

[2] See e.g. Field 1989, where it is suggested that "if the number 2 is a definite object, and the set $\{\varnothing,\{\varnothing\}\}$ is a definite object" there has "to be a definite question as to whether the former is the latter" (22)

[3] See e.g. Parsons 1990, where structuralism is characterized in part by the claim that "the objects [of which structuralism is true] have no more to them than can be expressed in terms of the basic relations of the structure" (303). See also Parsons 2004, especially the appendix.

The account I develop in order to achieve this goal holds that our ontology divides into disjoint categories, each with its own canonical way of determining the identity or non-identity of the objects it encompasses.[4] For instance, I argue that the identity or non-identity of natural numbers is determined by their positions in the number sequence, and the identity or non-identity of physical bodies, by facts having to do with spatiotemporal continuity. In this way we get truth-conditions for any intra-category identity statement. However, since natural numbers and physical bodies belong to different categories, the statement that 3 is identical with Caesar is not determined as true or false in any such way.[5] So the first set of intuitions—that there is something amiss with the question whether 3 is identical with Caesar—is correct when restricted to *canonical* grounds for the identity or non-identity of objects: no such grounds exist for either the identity or the non-identity of 3 and Caesar. However, I also argue that objects belonging to different categories are *ipso facto* distinct. Hence it follows that 3 is not identical with Julius Caesar. So the second set of intuitions is correct when *all* grounds for the identity or non-identity of objects are taken into account.[6]

When fleshing out this proposal, a number of difficult questions need to be addressed. What are the categories that I talk about, and what distinguishes them? Why must all cross-category identity statements be regarded as false? And how can knowledge of categories be ascribed to ordinary speakers and thinkers? I will approach these questions by reflecting on Frege's suggestion that reference to individual objects is mediated by sortal concepts, which provide criteria of identity for the objects referred to. Although many philosophers have found this suggestion intriguing, it has lacked a satisfactory interpretation.[7] I develop and defend a novel interpretation according to which

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[4] My account shares this general structure with the response to the Caesar Problem sketched in Wright 1983 and refined in Hale and Wright 2001*b*. But my account realizes this general structure in a rather different way.

[5] I will henceforth assume that persons belong to the category of physical bodies. My purpose in doing so is merely to simplify the discussion, not to make any controversial metaphysical claim. The reader is free to substitute his or her own favorite view of persons and adjust the argument of this paper accordingly.

[6] My discussion of mathematical structuralism in Linnebo 2003 invoked a more general distinction between what I called (in perhaps too Lockean-sounding language) the "primary" and the "secondary" properties of an object, the former being properties associated with the category to which the object belongs, and the latter being properties that the object has in virtue of belonging to this particular category. For instance, being odd and being prime are primary properties of the natural number 3, whereas being a natural number and being abstract are secondary properties.

[7] Some interpretations are found in Dummett 1981*a* and 1991, Evans 1982, Hale and Wright 2001*b*, Wiggins 2001, Williamson 1990, and Wright 1983.

reference has a richer structure than it is normally taken to have: There are certain canonical forms of reference which are mediated by a *referential intermediary* and guided by a *unity relation* that specifies when two such intermediaries determine the same object. For instance, canonical reference to a direction is mediated by a line (or some other directed object) and guided by the principle that two lines determine the same direction just in case they are parallel; and canonical reference to a body is mediated by a spatiotemporal part of the body in question and guided by the principle that two such parts determine the same body just in case they are spatiotemporally continuous in a certain natural way. I use this structure to give precise definitions of the important but slippery notions of criterion of identity and sortal concept. As will become clear, this yields the conclusion that to be is to be an F for some sortal F.[8] Although I believe Frege anticipated important aspects of this interpretation, it is doubtful he would agree with all of it. But my present goal is primarily systematic, not exegetical.

In the next section I begin by clarifying the claim that reference requires criteria of identity. In the following two sections I describe and defend my account of criteria of identity and sortal concepts. In the final section I use this account in an attempt to answer the questions about categories mentioned in the previous paragraph. Throughout, I rely on a broadly Fregean approach to meaning, where, following Frege, I distinguish between sense and reference. The sense of an expression is understood as the mode of presentation of its referent, and a complete understanding of an expression is identified with knowledge of its sense.[9]

2. Frege's Principle

In *Grundlagen* §62 Frege makes the following claim.

> If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all
> cases whether *b* is the same as *a*, even if it is not always in our power to apply this criterion.
> (Frege 1884, 73)

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[8] Hints of this view are found in Dummett 1981*a* and 1991. Perry 2002 and Williamson 1990, chapter 9, contain more explicit anticipations; but insofar as these anticipations are developed, it is in a theoretical framework that is different from mine.
[9] This broadly Fregean approach is shared in the works of Dummett and Evans cited in footnote 8. For a nice introduction to this approach, see Dummett 1978.

This passage—arguably one of the most important in the book[10]—introduces the notion of a criterion of identity and claims that in order to refer to an object a criterion of identity must be provided for the object in question.[11] I will call this *Frege's Principle*.

Frege's Principle is based on the idea that in order to use a symbol to refer to an object one needs to know what object the symbol is referring to. But this idea needs to be qualified if it is to be defensible. Firstly, the knowledge in question need not be verbalizable but could consist in a practical ability to identify the object in question, for instance when presented with it. Secondly, the idea does not apply to all uses of language. Assume I overhear the following conversation.

> First interlocutor: 'Bob is really important to our project.'
> Second interlocutor: 'Yeah, we need to find out more about Bob.'

Since this is all that I hear, I do not know which object Bob is; in fact, I do not even know whether Bob is a person, a dog, a planet, or even a natural number. Nevertheless, it seems I may later use the name 'Bob' to refer to whichever object the interlocutors were talking about when they used the name. What this example brings out is that language can be used parasitically. As long as a linguistic expression is a meaningful part of some shared language, I can successfully use this expression although I do not myself understand it. We must therefore restrict the idea in question to non-parasitic uses of language, that is, to uses where the speaker fully understands the expression in question. Thus qualified, the claim is that to fully understand a singular term one must know what object it refers to. This claim (which is essentially what Evans 1982 calls "Russell's Principle") is highly plausible.

Next, Frege proposes that in order to know what object a singular term **a** refers to, one needs to know what distinguishes its referent *a* from other objects.[12] So one needs to know what it is for an

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[10] Dummett 1991 goes so far as to call it "arguably the most pregnant philosophical paragraph ever written" (111).

[11] Frege appears to assume that it is *objects* that have criteria of identity. For instance, he talks of a "criterion for the identity of numbers" (*ibid*.) shortly after arguing that numbers are objects. This means that the passage just quoted is guilty of a use/mention confusion: The first occurrence of the sign '*a*' in the quoted text signifies a symbol, whereas the second signifies a non-linguistic object. However, we will see that the correct statement of Frege's idea is a rather subtle matter.

arbitrary object *x* to be identical with or distinct from *a*. I propose to analyze this knowledge as the state of understanding the open formula $\ulcorner a = x \urcorner$, where this understanding is taken to consist in knowledge which, for any singular term **b** that one might come to understand, would enable one to understand the sense of $\ulcorner a = b \urcorner$. I will call the sense of the open formula $\ulcorner a = x \urcorner$ a *criterion of identity* applicable to **a**. This places criteria of identity at the level of Fregean sense.[13] For to understand $\ulcorner a = b \urcorner$ is to know its sense, or, equivalently, to know the canonical truth-condition of this identity statement. But as remarked above, this knowledge need not be verbalizable. Nor does this knowledge need to be shared by everyone who understands the singular term **a**; different people may have different ways of distinguishing its referent from other objects. Frege's Principle does therefore not commit us to the strong claim that every singular term has a unique sense that everyone who understands the term associates with it.

Putting things together, Frege's Principle becomes: To fully understand a singular term **a**, one needs to possess a criterion of identity applicable to **a**. However, this principle is still rather programmatic, since little has yet been said about what criteria of identity are or what form they should take. In the remainder of this section I will discuss two views of criteria of identity and reject one of them. In the next section I will present a third view, which is the one I favor.

Since criteria of identity are so closely tied to identity statements, I begin with an important general constraint on how such statements are to be analyzed. Let **S** be a sentence in which some singular terms, say **a** and **b**, occur extensionally. It seems that the referents *a* and *b* of these singular terms must be directly involved in the canonical truth-condition of **S**, in the sense that names of *a* and *b* must occur extensionally in the characterization of this truth-condition in the meta-language. After all, since **S** is about the referents *a* and *b*, its truth or falsity will depend on how things stand with these objects. For instance, if **S** is the sentence 'Alice loves Bill', then its truth or falsity will be a

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[12] I will use boldface letters to range over syntactic entities and the corresponding italicized letters to indicate their referents.

[13] Compare Dummett 1981*a* and 1991. Contrast Hale and Wright 2001*b*, especially 368-70, where the criteria of identity appear to operate at some level of facts. It is unclear to me where Evans 1982 stands on this issue.

matter of how Alice feels about Bill, and its canonical truth-condition must therefore directly involve Alice and Bill. This motivates the principle that Truth-conditions Involve Referents:[14]

(TIR)   The truth-condition of a sentence **S** directly involves the referents of all singular terms that occur extensionally in **S**, in the sense that names of these referents must occur extensionally in the characterization of **S**'s truth-condition in the meta-language.

This is just a special case of compositionality.

Clearly, (TIR) constrains our analysis of the truth-conditions of identity statements. The easiest way to meet this constraint is by means of what I will call the *minimalist view* of criteria of identity. This view holds that the only criteria of identity to be had, and the only criteria of identity that are needed, are the trivial ones that an identity statement ⌜$\mathbf{a} = \mathbf{b}$⌝ is true just in case the referents are one and the same:[15]

(T-Triv)   ⌜$\mathbf{a} = \mathbf{b}$⌝ is true $\leftrightarrow$ ref($\mathbf{a}$) = ref($\mathbf{b}$)

Minimalism trivially satisfies the constraint imposed by (TIR). However, the minimalist analysis of identity statements is completely uninformative; as is the analysis of criteria of identity to which it gives rise. By operating with such uninformative analyses, minimalism denies the notion of a criterion of identity any significant explanatory role. So although minimalism does not strictly speaking contradict Frege's Principle, it is clearly not in its spirit. For as we have seen, Frege believes that criteria of identity have an important role to play in the explanation of reference.

Another option is to take the canonical truth-condition of ⌜$\mathbf{a} = \mathbf{b}$⌝ to have the form $R(a, b)$, where R is some relation other than identity which analyzes what it is for the referents a and b to be identical. I will call the resulting view of criteria of identity the *metaphysical view*.[16] The case of set

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[14] A more systematic defense can be found in Linnebo forthcoming.

[15] Lowe 1989 holds a minimalist view for some kinds of objects; as do most people who find criteria of identity irrelevant to the theory of reference, for instance Ayers 1997.

[16] The view goes back at least to Locke, who ponders what various identities "consist in." See Locke's *Essay*, Book II, Chapter XXVII. More recently the view is explicitly endorsed in Lowe 1989 and 1997. Hale and Wright 2001*b* too appears to hold this view for some kinds of objects (see especially 390).

identity is often used to illustrate this view, since the identity of two sets x and y appears to consist in their having the same elements:

(Id-Sets) $Set(x) \ \& \ Set(y) \rightarrow (x = y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y))$

More generally, the idea behind the metaphysical view is that, for any sort F of objects, identity of two Fs consists in their standing in some relation R_F (which is not just identity):

(Id-Mp) $\forall x \forall y[Fx \ \& \ Fy \rightarrow (x = y \leftrightarrow R_F(x, y)]$[17]

However, the metaphysical view of criteria of identity is even less attractive than the minimalist one. The minimalist analysis at least has the advantage of being natural and perfectly well understood; its only shortcoming is being less informative than one might have hoped for. But the metaphysical view introduces serious complications without, it seems to me, any serious gain. The complications have to do with the relation R_F. Is every kind of objects really associated with some such relation? If so, what are these relations? Moreover, even if these questions could be answered, it is not clear how helpful the metaphysical view would be to our project.[18] Since the singular terms 'a' and 'b' occur extensionally in '$R(a, b)$', the fact that $R(a, b)$ is the same as the fact that $R(a, a)$. But this means that what makes a identical with b is the same as what makes a identical with itself. For instance, what makes me, right now, identical with a little boy in 1975 is the same as what makes me right now identical with myself. But this deprives the notion of a criterion of identity of what was supposed to be one of its primary purposes.[19]

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[17] Principles of this form are sometimes called *one-level* criteria of identity; see e.g. Lowe 1989 and 1997 and Williamson 1990, chapter 9. But note that on this usage, a "criterion of identity" is something entirely non-linguistic, whereas my usage locates criteria of identity at the level of Fregean sense.

[18] But to be fair, advocates of the metaphysical view often use the notion of criteria of identity for different purposes than mine.

[19] A radical alternative to both the minimalist and the metaphysical view is to hold that certain kinds of singular terms are "semantically inert," with the result that (TIR) does not apply to them. An example is the "tolerant reductionist" view that Dummett 1991 suggests for singular terms purporting to refer to mathematical objects. According to this view, certain kinds of singular terms are contextually defined by rules that allow complete sentences in which such singular terms occur to be transformed into sentences in which they do not. This is not a *general* alternative to the two views just discussed; for clearly, not all singular terms can be semantically inert. So on this view, the class of singular terms will bifurcate into those that genuinely refer and those that are semantically inert. For further discussion, see Linnebo forthcoming.

3. A Meta-Semantic Interpretation of Frege's Principle

Let's take a step back and attempt to diagnose what has gone wrong. Both the minimalist and the metaphysical views characterize the truth-condition of an identity statement ⌜**a** = **b**⌝ in ways that involve the referents *a* and *b* themselves, fully individuated. So in order to understand these truth-conditions, we must *already* be capable of referring to the objects *a* and *b*. But was not Frege's Principle supposed to be part of an explanation of how such reference comes about in the first place? What Frege proposes seems to be that someone's understanding of a singular term consists (at least in part) in knowledge of a criterion of identity applicable to the term. But for this proposal to make sense, there must be at least *some* characterization of the truth-condition of an identity statement that does not presuppose an ability to identify the referents of the singular terms flanking the identity sign.

This reading of Frege's Principle is confirmed by the context in which the principle is first announced, namely *Grundlagen* §62. This section opens with the question "How, then, are numbers to be given to us, if we cannot have any ideas or intuitions of them?" Frege then suggests that "[s]ince it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs." Next Frege notes that since the numerals purport to denote objects, and since it is fundamental to objects that they can stand in the relation of identity, it becomes especially important to fix the truth-conditions of identity-statements. Immediately thereafter he gives the statement of Frege's Principle that I quoted at the beginning of the previous section. So the context leaves no doubt that Frege's Principle is supposed to play an important role in the explanation of how reference comes about, both to numbers and more generally.

But the principle (TIR) appears to impose an order of explanation that is antithetical to this suggestion. For when an identity statement ⌜**a** = **b**⌝ is understood in accordance with (TIR), then, in order to determine its truth-value, it seems one would first have to determine what objects the singular terms **a** and **b** refer to, and then determine whether these referents in fact stand in the relation of identity. But if so, knowledge of criteria of identity *presupposes* an understanding of reference, and thus cannot be used to explain it.

I believe the key to resolving this apparent impasse is to distinguish sharply between *semantics* and *meta-semantics*. Semantics is concerned with how the semantic value of a complex

expression depends upon the semantic values of its various simple constituents. To explain this, we ascribe to each complex expression a certain semantic structure and explain how its semantic value is determined by this structure as a function of the semantic values of its simple constituents. Meta-semantics, on the other hand, is concerned with what is involved in an expression's having the various semantic properties it happens to have. In particular, it attempts to explain in virtue of what an expression has its semantic structure and its semantic value.[20] With this distinction in mind, we see that (TIR) is a purely semantic principle: It assumes that the various simple expressions have been assigned suitable semantic values and uses this to say something about the canonical truth-conditions of complex sentences. On the other hand, Frege's proposal that knowledge of criteria of identity have an important role to play in the explanation of how reference comes about is concerned with meta-semantics.

Since (TIR) and Frege's proposal thus have different concerns, perhaps they do not conflict after all. Being a semantic principle, (TIR) applies to the canonical truth-conditions generated by a sentence's semantic structure. But perhaps semantic structure is not the only kind of structure that matters. Perhaps some additional kind of structure allows us to give a different characterization of the truth-conditions of identity statements which, although compatible with the canonical truth-condition, is more informative. When a determinate object is singled out in a successful act of reference, this process is presumably mediated by various devices. So presumably the senses of singular terms have some internal structure or articulation. But if so, this internal structure or articulation may make available entities other than the referents *a* and *b* in terms of which the truth-conditions of an identity statement ⌜**a** = **b**⌝ can be expressed.

If a view of this sort can be made out, it will provide an alternative to the minimalist and the metaphysical views of criteria of identity. I will refer to such an alternative as a *meta-semantic* view (without thereby implying that the view does not have metaphysical consequences). Can a meta-semantic view be developed and defended? A great number of powerful examples bode well.

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[20] This distinction between semantics and meta-semantics is essentially the same as that which Stalnaker 2001 draws between "descriptive" and "foundational" semantics.

(i)   *Directions*. Reference to a direction is mediated by a line (or some other directed object) that has the direction in question. For instance, I may point to a line or use my arm to indicate one and then refer to a direction by the demonstrative expression 'this direction'. An identity statement involving two such references to directions will be true just in case the two mediating lines are parallel.

(ii)   *Shapes*. This case is analogous to that of directions: Reference to shapes is mediated by a thing or a figure that has the shape in question, and an associated identity statement will be true just in case the two mediating things or figures are congruous.

(iii) *Syntactic types*. Reference to a syntactic type is mediated by one of its tokens. For instance, I may point to a printed letter and say 'this letter (type)'. An identity statement involving such references to letter types will be true just in case the two mediating tokens are of the same type.

(iv)   *Natural numbers*. Reference to a natural number is mediated by a member of some progression of numerals (considered as syntactic types). For instance, I may point to a printed numeral **n** and say 'this number', or I may simply *use* this numeral. In addition to the ordinary decimal numerals, there are numerals in binary notation, Roman numerals, and sequences of strokes. An identity statement involving two such references to natural numbers will be true just in case the two mediating numerals occupy analogous positions in their respective progressions.[21]

(v)   *Sets*. Reference to a set is mediated by a list of its elements.[22] Given two such lists, then the identity statement concerning the corresponding sets will be true just in case any object that figures on one list also figures on the other.[23]

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[21] This is an *ordinal* view of the natural numbers, according to which the numbers that figure in our ordinary thought and talk are individuated by their positions in the number sequence. Related ideas are found in Parsons 1971. This contrasts with the *cardinal* view advocated by logicists, according to which the natural numbers are finite cardinals, individuated by Hume's Principle (which says that two numbers are identical just in case the concepts whose numbers they are are equinumerous). I provide some evidence for the ordinal view in Linnebo 2004*a*. However, most of the arguments that follow could straightforwardly be adapted to the cardinal view.

[22] The lists in question must obviously be required to be of "set-size" length. They can thus be mathematically represented as functions from an ordinal to the universe. I intend to elaborate on this view of sets in future work.

[23] Note that this brings what appeared to be an example of the metaphysical view of criteria of identity under the umbrella of the meta-semantic view.

(vi)   *Physical bodies*. Reference to a physical body is mediated by a spatiotemporal part of the body in question. An associated identity statement will be true just in case the two mediating parts are spatiotemporally continuous in some appropriately natural way.[24]

More generally, Frege's claim is that canonical cases of singular reference are based on two elements.[25] First, there is a mediating entity in terms of which the referent is presented. I will refer to such entities as *referential intermediaries*. Second, there is a relation that specifies the condition under which two referential intermediaries determine the same referent. I will refer to such relations as *unity relations*. Like the referential intermediaries, a unity relation belongs at the level of reference, not at the level of language or Fregean sense. Finally, I will refer to an ordered pair $(u, \approx)$ consisting of a referential intermediary u and a unity relation $\approx$ applicable to this intermediary as a *referential attempt*. What Frege proposes is thus that canonical reference is based on referential attempts. Does this give a satisfactory (meta-semantic) explanation of how a singular term comes to refer?

Consider a singular term **a** that refers canonically. The proposal is that this reference comes about by the term's being associated with some referential attempt $(u, \approx)$ that specifies how the referent is presented and when two such presentations determine the same referent. Assume someone understands the term **a** and thus also knows that it is associated with the referential attempt $(u, \approx)$. This person thereby masters a criterion of identity applicable to **a**. For let **b** be any other singular term that the person may come to understand and that also refers canonically to an object of the sort in question. Then **b** too must be associated with a referential attempt, say $(v, \approx)$. Our subject will then grasp the following reduced truth-condition:

(T-Red)   ⌜**a** = **b**⌝ is true $\leftrightarrow u \approx v$

And someone who grasps this reduced truth-condition will be able to track the referent of **a** and to distinguish it from other objects of the same sort. This competence is naturally described as knowing

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[24] I elaborate on this view of reference to physical bodies in the next section.

[25] I will not here have anything to say about non-canonical reference. Following Michael Dummett and Gareth Evans, I believe non-canonical reference can and must be explained in terms of canonical reference. This explanation can for instance be based on someone's ability to recognize the referent of a singular term when presented with it in a canonical way. See Dummett 1981*a*, 231-239 and Evans 1982, 109-112.

what object **a** refers to. Consider for instance the case of physical bodies. Assume someone is digging in her garden, hits upon something hard with his shovel, and as a result forms the thought: This body is large. Later she hits upon something hard again, one meter away from the first encounter, and as a result forms the thought: This body is identical with that body. Finally, our subject appreciates that this identity is true just in case the two chunks of solid stuff that she has hit upon are spatiotemporally connected in some suitably natural way. It is extremely plausible to regard this capacity as a capacity to refer to physical bodies.

On this view, the unity relation $\approx$ implicitly defines a (partial) function f that maps a referential intermediary u to the referent, if any, that u determines. This is encapsulated in what I will call *principles of individuation*:

(PI) $\qquad\qquad \forall u \forall v \, (fu = fv \leftrightarrow u \approx v)$[26]

Of course, when formulating principles of individuation, we philosophers make use of our own ability to refer to objects of the sort in question. But this is permissible. We are allowed to presuppose *that* we can refer to objects of the kind in question. What we are not allowed to presuppose is an explanation of what this ability *consists in*. But no such presupposition is made.

I claim that this analysis of reference and individuation offers just what we need. Firstly, this analysis respects the semantic principle (TIR) because it takes the semantically generated truth-condition of an identity statement to be the following trivial one:

(T-Triv) $\qquad\qquad$ $\ulcorner$**a** = **b**$\urcorner$ is true $\leftrightarrow$ ref(**a**) = ref(**b**)

But secondly, this analysis shows how suitable meta-semantic knowledge enables one to grasp the more informative truth-condition (T-Red). It is particularly important that this reduced truth-condition can be grasped without any prior ability to pick out the referents of the singular terms **a** and **b**. It

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[26] Principles of this form are sometimes called *two-level* criteria of identity; see footnote 17. For two approaches based on such principles, see Perry 2002 and Williamson 1990, chapter 9. Logically speaking, principles of the form (PI) are the same as the so-called *abstraction principles* that have received so much attention from neo-logicists, see e.g. Wright 1983 and Hale and Wright 2001*a*. But the theoretical frameworks in which these principles are embedded are quite different. For a comparison, see Linnebo forthcoming.

suffices for grasp of this truth-condition that the subject knows that the singular terms are associated with the referential attempts $(u, \approx)$ and $(v, \approx)$. This makes the reduced truth-condition perfectly suited to the meta-semantic task of explaining how reference to the relevant sort of objects comes about. So here we really do have an alternative to the minimalist and the metaphysical views of criteria of identity.[27]

However, an adequate grasp of the reduced truth-condition (T-Red) does presuppose that the subject stands in some appropriate relation to the referential intermediaries u and v and to the unity relation $\approx$. What is this "appropriate relation"? In some cases the subject's relation to the referential intermediaries will be one of full-fledged reference. This reference must then be explained by invoking out meta-semantic account a second time. (An example of this is (iv) above, according to which reference to a natural number is mediated by a numeral *type*, reference to which is in turn mediated by a numeral *token*.) However, on pain of ill-foundedness, this cannot always be the answer; in particular, this cannot explain how reference comes about in the first place. There must be some basic form of reference which does not presuppose that the subject is already capable of any form of reference. In the next section I will discuss what I take to be the most important case of such basic reference, namely reference to physical bodies. I will suggest that here the subject's relation to the referential intermediaries is a purely causal one.

Another complication is that not every equivalence relation can serve as a unity relation in a principle of individuation. For instance, if we let the variables 'u' and 'v' range over concepts and let $\approx$ be the relation of co-extensionality, we will be able to derive Russell's paradox. So some account will be needed of which principles of individuation are acceptable and which are not.[28]

Finally, one may wonder whether it really is essential to regard the structure in terms of which I have argued that reference is canonically determined—a function f applied to a referential

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[27] This is also an alternative to the "tolerant reductionist" view of Dummett 1991 (mentioned in footnote 19). For on the Fregean view that I have defended, even mathematical terms have genuine reference. For further discussion, see Linnebo forthcoming.

[28] In the context of neo-logicism, the problem of distinguishing acceptable abstraction principles from unacceptable ones is known as "the Bad Company Objection." The neo-logicists have no satisfactory general solution to this problem. I am more optimistic about the corresponding problem for principles of unity, as such principles are embedded in a richer theoretical framework. For some preliminary discussion, see Linnebo forthcoming.

intermediary *u*—as meta-semantic rather than semantic. Could we not avoid introducing this new kind of structure and make do with the familiar notion of semantic structure? I will now point to some independent linguistic evidence that the answer to this question is *no*. Had I claimed that this structure is semantic, my proposal would have been akin to the descriptive theory of names. But that proposal would have been implausible for at least two reasons. Firstly, in the semantics of natural language, a function-argument structure is typically achieved by means of a definite description: the *f* of *u*. Two examples are 'the capital of France' and 'John's advisor'. But definite descriptions are non-rigid, whereas the singular terms we have considered all appear to be rigid. For instance, the singular term 'this shape' [pointing to a piece of clay shaped as a sphere] refers to a certain spherical shape even in worlds in which the piece of clay is shaped as a cube. (This is what allows us to say truly: This piece of clay might not have had this shape.) Secondly, expressions that are semantically structured as a function applied to an argument allow existential generalization into this argument place. For instance, from 'The capital of France is beautiful' we may infer that there is a country whose capital is beautiful. But no such existential generalization is permitted on the singular terms we have considered. For instance, from 'This letter (type) is beautiful' [pointing to a letter token] or 'This body is large' [pointing to some part of a body] we cannot infer that there is some token whose type is beautiful or that there is some spatiotemporal part whose associated body is large. By holding that the postulated structure is meta-semantic, I avoid these two problems. For the meta-semantic facts in virtue of which a singular term has its reference have no immediate bearing on questions of rigidity; nor need they inform people's inferential dispositions in the way that semantic facts do.

This is not to deny that in a lot of cases the postulated structure can be *made* semantically available. For instance, instead of referring to a particular shape by means of the singular term 'this shape' [pointing to a clay sphere], I may refer to it descriptively as 'the shape of this piece of clay'. As usual, this description is non-rigid: In worlds where the piece of clay was shaped as a cube rather than a sphere, the description would be true of some cubic shape rather than of a spherical one. (Giving the description narrow scope, it is therefore false to say: This piece of clay might not have had the shape of this piece of clay.) Moreover, existential generalization will allow us to make an

inference from 'The shape of this piece of clay is spherical' to 'There is something whose shape is spherical'.

4. Minimalism and Reference to Physical Bodies

In Section 2 I rejected the metaphysical view of criteria of identity, and in the previous section I argued that the meta-semantic view enjoys a great deal of plausibility. But I have so far had little to say about the minimalist view, according to which the only criteria of identity to be had, and the only criteria of identity that are needed, are the completely uninformative ones that ⌜$a = b$⌝ is true just in case the referents *a* and *b* are one and the same. I will now argue that minimalism fails to provide an adequate account of reference to physical bodies. Since this example is extremely important, both in its own right and as a source of minimalism's intuitive appeal, my argument will, if successful, leave minimalism seriously weakened.

One of the most appealing features of minimalism is that it appears to offer a correct analysis of reference to physical bodies. For it seems that such reference takes place without any referential intermediaries or unity relations. Assume you are sitting in front of a box full of small rocks. It seems that you can single out one of these rocks simply by grabbing hold of it! It seems that the world itself will determine how large the rock is that you have grabbed hold of, whether it is the same rock as the one I have just grabbed hold of, and whether it is the same as the one you grabbed hold of yesterday, thus leaving no need for a referential intermediary or a unity relation. In fact, the "grabbing" in question need not even be understood literally. It suffices to make perceptual contact with a rock or in some other way to specify a spatiotemporal part of one.

The reason why it is so easy to refer to rocks appears to be that rocks are "natural units" that have been "carved out" by reality itself. In contrast, it is much harder to refer to "unnatural" objects such as the lower half of a certain rock, the mereological sum of it and another rock, or one of its one-second temporal parts. Since the world itself does not "carve out" such gerrymandered objects, the option of singling one out simply by grabbing hold of it is not available. Rather, these objects can be singled out for reference only by means of certain concepts. For instance, in order to refer to the lower half of a rock, you need to know the difference between upper and lower; to refer to the mereological

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sum of it and another rock, you need to know some mereology; and to refer to arbitrary time-slices of it, you need some knowledge of time.

Generalizing, one may attempt to use this intuitive notion of naturalness to motivate the metaphysical view that reality consists of certain privileged natural objects, and that unnatural objects, insofar as such exist at all, are composed of natural parts. It can for instance be argued that half-rocks and mereological sums of rocks are composed of physical particles, which are natural units. Moreover, if the existence of one-second rock-stages is granted at all, these objects can plausibly be taken to be composed of particle-instants. On proposals such as these, "the furniture of reality" consists of basic objects that are natural, as well as arbitrary mereological sums of such basic objects, some of which are natural and some not. Minimalism would then hold of all basic objects and perhaps of some non-basic ones.

But I deny that there is any defensible notion of naturalness capable of underpinning minimalism in the way suggested in the previous two paragraphs. To defend this claim, I will now consider reference to physical bodies in some detail, as this is the case where minimalism and the notion of naturalness appear to be at their strongest. I will argue that, contrary to appearances, reference to a body does require a referential intermediary and a unity relation. If my argument succeeds, minimalism will be deprived of what appeared to be one of its strongest and most important applications. I will begin by explaining why we need not be aware of way in which reference to physical bodies is structured. Then I will explain how I believe such reference is in fact structured.

When referring to bodies, we are not aware of any of the structure that the meta-semantic view postulates. For bodies are given to us in perception *as such*. For instance, if I close my eyes and quickly reopen them, I almost immediately perceive an arrangement of bodies; in this particular case a laptop computer, a desk lamp, a few pens, and some books, arranged against the background of the surface of my desk and a window. So perception here presents me with a scene composed of well-distinguished and unified bodies arranged against a more neutral background. There is nothing I consciously have to do in order to apprehend this arrangement of bodies. No effort is required. In fact, even if I try, I am not able to perceive the scene otherwise. Other kinds of objects are not perceptually given to me as unified objects. Consider for instance unusual objects such as the left half of my

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computer or the sum of my computer and my desk lamp. Although I can in a sense perceive these objects, it requires a certain effort to do so: I must actively and consciously bring certain concepts to bear. For instance, in the case of the left half of my computer, I need the concepts of *half* and of *right* and *left*.

However, these phenomenological observations do not contradict the meta-semantic view. We must not conflate the absence of *conscious* conceptualization with the absence of conceptualization *of any sort*. The observation that bodies are given to us as natural units establishes only the weaker claim that our apprehension of bodies does not require any *conscious* conceptualization. But we cannot infer from this that our apprehension of bodies involves no conceptualization or structuring *at all*. It remains a possibility that our perception involves some *unconscious* form of conceptualization or structuring.[29] This is in fact what a lot of contemporary psychologists think.[30] I believe this distinction between objects that are perceptually given to us as unified objects and objects that are not goes a long way towards explaining the intuitive contrast between "natural" and "unnatural" objects.[31]

To find out how reference to bodies may be structured, it is useful to begin by asking what it would take for a robot, equipped with capacities similar to the natural human ones, to individuate and refer to bodies.[32] So assume we have a robot that interacts with its environment by detecting light reflected by surrounding surfaces, as well as by touching and grasping stuff in its vicinity. I will begin with a thumbnail version of my argument. To refer canonically to a body, the robot will have to "perceive" it in the sense of receiving light from some part of its surface or touching some part of it. The robot will thus have to causally interact with some spatiotemporal part of the body. These parts need not have natural boundaries in either space or time; they are simply the sum-totals of the particle-instants with which the robot causally interacts in a perception-like way. This provides a class

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[29] Here I differ from Dummett, who holds that criteria of identity are learnt and involve linguistic understanding. See Dummett 1981*a*, 498, 505, and 573.

[30] See e.g. Spelke 1993 and Xu 1997.

[31] But not all the way: see Section 5.

[32] I focus on the senses of sight and touch, leaving out smell and hearing, which appear to play little role in the individuation of bodies.

of referential intermediaries. Next, we need to characterize the conditions under which two such referential intermediaries determine the same body. Assume for instance that the robot establishes perceptual contact with part of a rock that sticks out of the ground and that one of its "arms" is simultaneously probing into the ground nearby and encountering something hard. What should we "teach" the robot about the conditions under which these two referential intermediaries determine the same body? The answer will have to do with the two parts' being spatiotemporally continuous in some appropriate way.

This argument must now be fleshed out. I begin with the problem of the nature of the subject's relation to the spatiotemporal parts that serve as referential intermediaries. It is problematic to regard this relation as full-fledged reference. As we have seen, it does not *seem* right that reference to physical bodies is mediated by reference to any other sort of object. Moreover, if we always needed to *refer to* the referential intermediaries, we would have a vicious regress. For what intermediaries would this reference be based on? And how would we refer to these intermediaries in turn? It is much more plausible that the subject's relation to the spatiotemporal parts that serve as intermediaries is a purely causal relation, not one of full-fledged reference.[33]

I now turn to the relation of natural spatiotemporal continuity that has to hold between two parts for them to determine the same body. Roughly, the kind of continuity that matters has to do with solidity and motion: The two parts must be related through a continuous stretch of solid stuff, all of which belongs to the same unit of independent motion. More details are spelled out in the following five characteristics of bodies, which I take to be part of an analysis of the concept.[34]

(B1)   Bodies are three-dimensional, solid objects.

This characteristic holds of bodies because any two parts of a three-dimensional solid are spatially related in the appropriate way. It is also the case that two distinct bodies cannot occupy the same

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[33] Here I again differ from Dummett, who attempts instead an analysis based on Strawsonian feature-placing sentences. See Dummett 1981*b*, 217 and 1991, 162f.

[34] These characteristics are also constitutive of psychologists' concept of a so-called "Spelke-object," which corresponds closely to my concept of a physical body. See the references of note 30.

region at the same time.[35] (To see this, assume for *reductio* that x and y are bodies occupying the same region at the same time. Then the spatial parts of x and y would be so related as to ensure the identity of x and y.)

(B2)   Bodies have natural and relatively well distinguished spatial boundaries.

For instance, a half-rock fails to be a body because it lacks sufficiently natural boundaries, and a mountain fails because its boundaries are insufficiently well distinguished.

(B3)   Bodies are units of independent motion.

Thus, although a book is a body, a pile of papers is not. In fact, the pile of papers would not be a body even if the sheets were piled so densely that there was no air between them but only solid paper throughout. This shows that (B3) is independent of (B2).

(B4)   Bodies move along continuous paths.

Consider the object that came into being with the birth of Bill Clinton, coincided with Clinton until the end of his presidency, and thenceforth coincides with George W. Bush. By (B4), this object does not qualify as a body.

(B5)   Bodies have natural and relatively well distinguished temporal boundaries.

So arbitrary temporal parts of bodies are not themselves bodies. Note that it follows from (B2) and (B5) that bodies have unobserved spatial and temporal parts. This means that bodies exist independently of our perception of them.

Whether or not this particular analysis is complete and correct in all of its details, it provides strong evidence that there is some concept that guides our individuation and re-identification of

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[35] Since my claim is restricted to bodies, it is compatible with the claim that a statue is numerically distinct from the lump of clay of which it is made. The reasons offered for distinguishing between the statue and the lump turn on regarding either the status or the lump as something other than a body, either the statue as essentially a statue or the lump as essentially a sum of certain atoms.

bodies. The first three characteristics of my analysis are important in the individuation of bodies, and the last two in their re-identification.

5. Sortals and Categories

The overarching goal of this paper is to defend the view that our ontology divides into disjoint categories, each with its own canonical way of determining the identity or non-identity of the objects it encompasses. I will now use the ideas defended in the previous three sections to flesh out the view. Recall from Section 1 the questions about this view that need to be addressed: What are these categories, and what distinguishes them? Why must all cross-category identity statements be regarded as false? And how can knowledge of categories be ascribed to ordinary speakers and thinkers?

Let's say that a predicate **F** is a *sortal* if its sense is associated with some particular type of referential intermediary and some unity relation that specifies when two such intermediaries determine the same object. This definition accords well with the more common definitions of sortalhood. A common informal characterization says that a predicate **F** is a sortal if use of it in predication calls for the indefinite article. For instance, since we say 'Bob is a cat' but not 'Bob is a heavy', this characterization implies that the predicate 'cat' is a sortal but that 'heavy' is not. A common, more precise definition says that a predicate **F** is a sortal if it allows for meaningful questions about identity and numerosity.[36] For instance, it makes sense to ask whether this cat is identical with that cat or how many cats there are in this room. In contrast, if we replace the predicate 'cat' with the predicate 'heavy', the corresponding questions make no sense. So again it turns out that 'cat' is a sortal but 'heavy' not. My definition of sortalhood subsumes and explains these more common characterizations. The meaningfulness of identity contexts is a necessary condition for sortalhood because a sortal is associated with a unity relation, which provides grounds for the identity

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[36] Definitions of this general sort are found in Hale and Wright 2001*b,* Lowe 1997, and Wright 1983, 2. Deriving from Strawson 1959 there is also a more Aristotelian conception of sortalhood, which requires in addition that a sortal concept be an answer to the Aristotelian 'What is it?' question and thus be such that, if it applies to an object, it cannot cease to do so; for instance, a horse allegedly cannot cease to be a horse. See also Dummett 1981*a*, 76 and Wiggins 2001, 8-11. I make no attempt to capture this Aristotelian requirement.

and non-identity of the objects in question. The meaningfulness of most[37] questions about numerosity is a necessary condition for much the same reason.

Next, let's say that a sortal is *maximal* if there is no other sortal, associated with the same type of referential intermediary and the same unity relation, that is more inclusive. A maximal sortal is thus a predicate which is true of every object that can be referred to by means of a certain type of canonical referential attempt. Examples of maximal sortals include predicates that we intuitively take to be "category words," such as 'direction', 'natural number', and 'physical body'. Let's therefore define a *category* as any class including all and only those objects of which a maximal sortal is true.

Let an *intra-category identity statement* be any sentence of the form ⌜**a** = **b**⌝ where the singular terms **a** and **b** are associated with referential attempts based on one and the same unity relation. Any intra-category identity statement is determined as true or false in the canonical way specified by this shared unity relation. Let a *cross-category identity statement* be any sentence of the same form but where the two referential attempts involve different unity relations. An example is the sentence '3 = Julius Caesar' with which the paper began. Such identity statements are not determined as either true or false in any canonical way. So our intuition that there is something flawed or illegitimate about the question whether the natural number 3 is identical with Julius Caesar is correct when restricted to *canonical* grounds for the identity or non-identity of objects.

What *are* we then to say about cross-category identity statements? I will now argue that the most attractive view is that such sentences are just false. To begin with, this view is appealingly simple. For all it does is add to what we have already said about how identity statements are determined as true an "extremal clause" to the effect that these are the *only* ways in which identity statements are determined as true. Specifically, the view insists that the *only* way in which two referential attempts can determine the same object is by being associated with some shared unity relation, by which the two intermediaries are related. This allows us to express criteria of identity in

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[37] Arguably there are kinds of objects such that, although grounds have been provided for the identity or non-identity of any given pair of such objects, it is impossible to assign any determinate number to the totality of such objects, if such a totality exists at all. For instance, although the principle of extensionality allows us to identify and distinguish any given pair of sets, one arguably cannot count the number of sets.

an appealingly simple way. Let **a** be a singular term associated with some successful referential

attempt $(u, \approx)$. The criterion of identity applicable to **a** is then represented by the truth-condition

$$\ulcorner \mathbf{a} = x \urcorner \text{ is true iff } \exists v \, (x = fv \ \& \ u \approx v),$$

where f is the function implicitly defined by the unity relation $\approx$.

Moreover, this simple view coheres well with our intuitions. If asked whether the natural

number 3 is identical with the person Julius Caesar, the natural answer is *no*. Natural numbers and

persons are not the sorts of objects that *can* be identical, as their identity is governed by completely

different considerations. To say that a unity relation governs the identity of some sort of objects is to

make a strong claim about *the nature* of these objects. For instance, it says a lot about the nature of

natural numbers and physical bodies that the former are individuated by their position in a

progression, and the latter, by considerations involving natural spatiotemporal continuity. Natural

numbers are position-in-a-progression-individuated objects, whereas physical bodies are natural-

spatiotemporal-continuity-individuated objects. Since these are incompatible properties, no natural

number can be identical with a physical body.[38]

On the view I have just defended, no two categories can overlap. From this it follows that

objects "inherit" certain essential properties from the category to which they belong: It is essential to

an object that it be related to a referential intermediary of a certain type and that a certain principle of

individuation be valid for it. For instance, it is essential to a body that it have spatial parts and that it

be a spatiotemporal continuant, and it is essential to a natural number that it occupy some particular

position in a progression and that it be an object individuated by its position in this progression. These

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[38] It may be objected that 3 belongs both to the category of natural numbers and to that of reals. However, I believe this proposed counter-example is based on a mis-diagnosis of the fact that the numeral '3', as it occurs in ordinary language, is sometimes associated with one referential attempt and sometimes with another. But all this means is that this numeral is ambiguous, not that one object can belong to two categories.

Some harder cases involve artificial unity relations obtained by slightly modifying the ones in standard use. For instance, let a *p-body* be individuated just as a regular physical body except that its unity relation also requires that some contingent proposition *p* be true. Can a *p*-body also be a body? The answer will depend on how unity relations themselves are individuated. If they are individuated extensionally, then in any world where *p* is true, every *p*-body will also be a body. If on the other hand unity relations are individuated more like properties, then my view implies that no *p*-body (if there are any at all) can be a body. I am inclined towards the latter view, on the ground that I think the referents of our concept-expressions in general are more like properties than like sets.

properties are essential to the object in question because it cannot "be the object it is" unless it possesses these properties. To see this, let **a** denote the object in question. Then no identity statement of the form ⌜**a** = **b**⌝ can be true unless **b** is associated with the same sort of referential intermediary and the same unity relation as **a** is associated with.

The one question from our list that remains is how knowledge of categories can be ascribed to ordinary speakers and thinkers. In order to answer this question, it is important to bear in mind the order in which we have proceeded: first we defined the semantic notions of a sortal and a maximal sortal, then we defined the metaphysical notion of category in terms of these. If this approach is defensible, knowledge of categories is already implicit in the machinery that ordinary people use to refer to individual objects.

However, one may worry that this approach makes our definition of a category too epistemic. It may seem that our definition illegitimately projects into reality our contingent human modes of identifying and distinguishing between objects. Why should the fact that we human beings find it convenient to use certain concepts to organize our experience be any reason to think that reality is structured in accordance with these concepts? I will close by considering three objections that attempt to make this rather vague worry more precise.

The first objection is that my approach confuses semantic and metaphysical notions. But this simply is not so. The role of a sortal and its associated principle of individuation is merely to determine what kinds of objects we are talking about: namely those for which this principle of individuation is appropriate. For instance, directions are those objects reference to which relies on directed objects (such as lines) as intermediaries and on parallelism as a unity relation; and bodies are those objects reference to which relies on spatiotemporal parts as intermediaries and natural spatiotemporal continuity as a unity relation.

The second objection is based on Frege's rejection in *Grundlagen* §67 of a proposal that is structurally somewhat similar to mine. The proposal Frege rejects is that the concept of direction can be defined by stipulating that "q is a direction if it was introduced by means of the definition" that '$d(l_1) = d(l_2)$' is to be true just in case l_1 and l_2 are parallel. Frege objects to this proposal that a "definition of an object does not, as such, really assert anything about the object, but only lays down

the meaning of a symbol." For this reason, Frege says, we cannot treat the way in which an object is introduced as a property of the object; in fact, any object can be introduced by any number of different definitions. Although this objection is surely devastating against its intended target, it does not threaten my proposal. For the relation between a symbol and its referent is very different from that between a referential attempt and the object it picks out. The relation between a symbol and its referent is entirely contingent. Tradition and convenience aside, any symbol can be used to denote any object. In contrast, the fact that an object can be picked out by a certain type of referential attempt says a great deal about the object. In fact, since the object could not have been picked out by any other type of referential intermediary or any other unity relation, these are essential properties of the object.

The third objection is that my proposal is unable to recognize that certain kinds of objects, such as bodies, really are natural in a way that other kinds of objects are not. In order to deal with this objection, it is important to keep in mind that what I rejected in Section 4 was only a certain philosophical gloss on the notion of naturalness: I denied that the naturalness that bodies enjoy should be understood as some general metaphysical characteristic, guaranteeing that any object that possesses it can be referred to without the need for a referential intermediary or a unity relation. I argued instead that the proximal reason why bodies seem natural to us is that our perception of bodies is structured—without our being aware of it—in accordance with the unity relation appropriate to bodies, and thus that our perception highlights bodies rather than other kinds of objects. But the mere fact that our perception of bodies is structured in this way does not mean that the naturalness of bodies, when properly understood, is any less real or objective.[39] I agree that it would be a bad idea to deny that there is some sense in which bodies really are natural. After all, bodies are distinguished from their environment in a fairly sharp way, and this will presumably be an important part of an evolutionary explanation of the fact, to which I just appealed, that our perceptual system is structured in a way that privileges bodies.

My positive view on the naturalness that bodies enjoy is that it is *spelled out by* the concept of a body: It is the kind of naturalness that is characterized by a concern for spatiotemporal boundaries

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[39] For an example of a philosopher who thinks otherwise, see Ayers 1997, 395, where the "natural or real" is contrasted with the "conceptual or ideal."

and units of motion. Relative to this concern, there is a perfectly objective sense in which bodies are more natural than half-bodies, body-pairs, and one-second body-stages. For instance, if you are designing a robot to manage in the kinds of environment that humans typically inhabit, the concept of a body will be one of the most important concepts with which the robot will have to be equipped. Moreover, in order for the robot to refer to any of the gerrymandered objects mentioned above, it will *first* have to be able to refer to ordinary bodies and *then* acquire certain additional concepts involving space, time, or mereology.

So my meta-semantic approach to the metaphysical notion of a category appears to be defensible.[*]

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