Ontology and the Concept of an Object

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When people deny that there are objects of a certain kind, they normally take this to be a reason to stop speaking as if such objects existed. For instance, when atheists deny the existence of God, they take this to be a reason to stop speaking about God's will or His mercy. Or, to take a more mundane example, when people deny that there are round squares or that there are unicorns, they take this to be a reason to stop speaking as if rounds squares or unicorns existed.

Recent philosophy offers some interesting exceptions to this pattern. Philosophers frequently make the surprising move of rejecting some class of entities while simultaneously acknowledging that we have to go on speaking as if such entities existed. This move is motivated by an apparent tension between two sets of considerations. On the one hand, certain kinds of object are found to be philosophically problematic. Examples include mathematical objects, mereological sums, possible worlds, and events. This inclines philosophers to deny that such objects exist. On the other hand, there are discourses committed to these kinds of object which are not only logically coherent but serve important purposes. This inclines philosophers to preserve these discourses. There appears, in other words, to be a tension between ontological considerations, which favor the rejection of certain kinds of object, and other considerations, which favor the preservation of discourses committed to objects of these kinds.

A lot of recent philosphers take this apparent tension to be genuine and therefore attempt to do justice to both sets of considerations. Although they deny that there exist objects of the kinds in question, they grant that there is a sense in which statements committed to such objects nevertheless can be said to be correct. To spell out this sense of correctness, these philosophers typically appeal to some form of fictionalism.[1]

In this paper I argue against such fictionalist responses and propose what I take to be a better response. My argument is based on an examination of the central concept of ontology, namely the concept of an object. I deny that we possess a concept of object of the kind that is required to make sense of the fictionalist responses. For instance, I deny that we are able to make sense of the claim that arithmetic is perfectly in order logically but that there nevertheless are no numbers. Instead, I argue that the concept of object we do possess allows for objects that are sufficiently "thin" to dissolve the apparent tension. To illustrate this novel view of objecthood, I sketch a view of mathematical objects that takes their existence to consist in the logical coherence of the theory that describes them.

1. *The Fundamental Tension*

I will begin by examining in more detail the apparent tension between "bad" objects and "good" discourses committed to such objects. (Henceforth, I will refer to this as *the fundamental tension*.) I will use the natural numbers as my example. There are lots of other examples of the fundamental tension as well. But since these examples aren't fundamentally different from ours, little generality is lost by focusing on the natural numbers. First, I will explain why the numbers are felt to be philosophically problematic. Then, I will explain why even the nominalists, who deny the existence of numbers, nevertheless acknowledge that arithmetical discourse has desirable properties that make it worthy of preservation.

Almost all philosophical worries about the natural numbers have to do with their abstractness. The most common worry is epistemological. Because numbers are abstract, they are causally inefficacious. Philosophers therefore worry that numbers will be epistemologically inaccessible. The following passage from Hartry Field gives a characteristic example of this line of thought.

> According to the platonist picture, the truth values of our mathematical assertions depend on facts involving platonic entities that reside in a realm outside of space-time. There are no causal connections between the entities in the platonic realm and ourselves; how then can we have any knowledge of what is going on in that realm? ... It seems as if to answer these questions one is going to have to postulate some *aphysical connection*, some *mysterious mental grasping*, between ourselves and the elements of this platonic realm. (Field (1989), p. 68)

A second worry is based on the observation that pure mathematical objects, such as the natural numbers, appear to have no properties other than those they have in virtue of being positions in the structures to which they belong. Pure mathematical objects therefore seem to be "incomplete" in a way that distinguishes them from ordinary concrete objects. This prompts Field to ask the following rhetorical question: "if the number 2 is a definite object, and the set {∅, {∅}} is a definite object, doesn't there have to be a definite question as to whether the former is the latter?" (*ibid*., p. 22). Needless to say, Field does not expect the platonist to have a satisfactory answer.

Finally, a third worry, which is less often articulated but at least as influential, is that positing a large number of mathematical objects would somehow be metaphysically extravagant, and that it would therefore be wise to limit one's mathematical ontology as far as possible. This worry too is nicely expressed by Field:

> Admittedly, we can't have *direct evidence* against mathematical entities. We also can't have direct evidence against the hypothesis that there are little green people living inside of electrons and that are in principle undiscoverable by human beings; but it seems to me undue epistemological caution to maintain agnosticism rather than flat out disbelief about such an idle hypothesis. (*ibid*, p. 45)

Motivated by these worries, the nominalists deny the existence of mathematical objects. *A fortiori*, they deny that there exist prime numbers between 10 and 20.

However, this contradicts the familiar arithmetical theorem that such primes exist. This theorem is endorsed by professional scientists as well as by lay people. The nominalists find it slightly embarrassing in this way to be contradicting both science and common sense. They recognize that arithmetic is a serious enterprise with lots of desirable features. Firstly, they recognize that arithmetical discourse is associated with *strict standards of correctness*. There are clear and objective criteria for when it is correct to assert an arithmetical statement. Secondly, there is no serious doubt that these standards of arithmetical correctness are *logically coherent*. No contradictions can arise from making moves warranted by these standards. And thirdly, these standards are *tremendously useful* for everything from quotidian reasoning to advanced science.

So the nominalists realize they cannot reject arithmetical discourse out of hand. They cannot do to arithmetical discourse what atheists do to theological discourse, namely to scrap it altogether and offer nothing in its stead. In fact, most contemporary nominalists are sufficiently moved by the arguments just rehearsed to grant that arithmetical discourse ought to be preserved *as far as possible short of reaffirming the existence of numbers*. In granting this, they recognize a radical form of the fundamental tension.

In general, the fundamental tension arises because there are two conflicting sets of standards for assessing existence claims (say, the claim that there are *F*s). On the one hand, there are the ordinary standards that are local to the discourse in question (say, *F*-discourse). These standards tell us that *F*s exist. On the other hand, there are some stricter, philosophical standards, according to which *F*s are problematic and hence should not be allowed into our ontology. But if there are no *F*s, then all sentences of *F*-discourse that are committed to *F*s are false.

Can the fundamental tension be resolved simply by rejecting one of the sets of standards that conflict? As is well known, Rudolf Carnap argued that philosophical standards should be rejected in favor of ones that are local to the discourse in question. However, as W.V. Quine has taught us, there is no principled distinction between "external" philosophical questions and "internal" scientific ones. The sorts of considerations that are relevant to philosophical questions aren't essentially different from those relevant to the adjudication of the more general and abstract questions we encounter in science. So this strategy does not seem promising.

Other philosophers may be tempted by the opposite strategy of rejecting the ordinary standards in favor of the philosophical ones. However, we have granted that *F*-discourse is logically coherent and extremely useful in ordinary thought and in science. It would therefore be great philosophical arrogance to insist that this discourse be scrapped. Even a modicum of respect for common sense and science makes it hard to dismiss altogether a set of standards that is both coherent and very useful.

2. *Two-Status Approaches to the Fundamental Tension*

Moved by arguments of this sort, most philosophers who discuss the fundamental tension grant that the conflict is genuine and that both sets of standards have some validity. This leaves them no choice but to grant that there is something right about both sets of standards. Their goal is thus to reject *F*s while preserving *F*-discourse as far as possible short of reaffirming the existence of *F*s.

The strategy they pursue in order to achieve this goal is to distinguish two types of status that statements from the relevant class can enjoy. The highest status is *strict and literal truth*. Because of the philosophical worries about *F*s, statements committed to such entities are barred from this most dignified status. But there is also a second status, which falls short of strict and literal truth but is still an honorific. This second status is supposed to allow us to do justice to the standards that are local to *F*-discourse. Although claims committed to *F*s cannot *strictly speaking be true*, they can nevertheless be *correct* in the sense that they possess this second honorific status. The hope is that this will allow us to continue engaging in *F*-discourse without committing ourselves to *F*s. For ease of reference, I will henceforth refer to the two types of status as *Truth* and *Correctness*.

Obviously, this characterization of the two-status approach is purely programmatic. So if the approach is to be viable, it needs to be fleshed out. First and foremost, it needs to be explained what Correctness is. In particular, given that this status falls short of Truth, it must be explained why it is still an honorific. Why should participants in the relevant discourse strive to make only statements that are Correct, given that Correctness is no guarantee of Truth? Second, the distinction between Truth and Correctness needs to be connected up with our actual linguistic practice. It needs to be shown that both Truth and Correctness have some role to play in the relevant discourse and that these roles are distinct.

To answer the first question, philosphers typically appeal to some form of *fictionalism*. The basic idea is that, although claims committed to the problematic entities aren't strictly and literally true, they are nevertheless correct according to the relevant fiction. I will now examine two fictionalist proposals from the philosophy of mathematics. (Examples from other areas would have been possible as well. Fictionalist interpretations have been defended of, among other things, discourse about possible worlds, about mereological sums, and about events. But I have chosen two examples from the philosophy of mathematics because I believe this is the area where two-status approaches have their strongest appeal.)

The first example is the view defended by Hartry Field.[2] This view advocates a two-status approach based on the distinction between what is strictly speaking true and what counts as correct according to some set of useful but not necessarily truth-conferring standards. *Strictly speaking*, Field says, there are no mathematical objects, and most mathematical statements are therefore either false or vacuously true. Field is thus an error theorist about mathematics. But he grants that, *loosely speaking*, claims of platonistic mathematics can be correct or incorrect. They are correct if they follow logically from our standard mathematical theories, and incorrect otherwise.

If these standards of mathematical correctness provide no guarantee of Truth, what then *is* their significance? Why should people bother to ensure that their mathematical statements conform to these standards? Field's answer has to do with the practical utility of thus conforming. He argues that it is in principle possible to dispense with platonistic mathematics in favor of purely nominalistic formulations of our physical theories. However, it is extremely cumbersome to deduce consequences of these nominalistic theories. But this task would be greatly simplified if we were allowed to use platonistic mathematics. Fortunately, Field claims, platonistic mathematics can be added to these nominalistic theories at no cost. For although platonistic mathematics simplifies the deduction of consequences and provides a useful heuristic, it can always in principle be eliminated: Whenever a sentence in the nominalistic language follows from the nominalistic theory supplemented with platonistic mathematics, it can be shown to follow from the nominalistic theory alone. (Technically speaking, platonistic mathematics is *semantically conservative* over the nominalistic theories.) So platonistic mathematics is an innocent tool of great practical utility. And this gives us reason to conform to its standards.

The second example is a view recently developed by Stephen Yablo.[3] This view defends a different two-status approach, based on the distinction between what is *literally* true and what is only *figuratively* true. Literally speaking, Yablo claims, there are no mathematical objects. However, most ordinary, non-philosophical talk about mathematical objects is not intended to be understood literally. It is intended to be understood figuratively, much as people understand metaphors. Imagine, for instance, I say of someone that he has an ax to grind. Since I am only speaking metaphorically, I am not committing myself to any tools in his possession. Likewise, Yablo suggests, when we say that the average star has 2.4 planets, we are not committing ourselves to any numbers; we are merely exploiting number-discourse to construct a *mathematical metaphor* that allows us to make a statement about the physical world that it would otherwise have been hard or impossible to make.

The second question that confronts a two-status view is how to integrate this view with our actual linguistic practice. For most uses of language, a two-status view is not very natural. As noted at the beginning of this paper, when we deny that there exist objects of some sort, we normally take this to be a reason to stop speaking as if such objects existed. To speak otherwise would not only be false but incorrect by *any* ordinary standard. So to defend a two-status interpretation of a certain discourse, it needs to be explained how this discourse differs from ordinary uses of language in a way that allows two different standards to exist side by side.

For some kinds of language use, this task can no doubt be carried out. For instance, when metereologists talk about centrifugal forces (as I am told they often do), there is a clear sense in which they aren't committing themselves to such forces but merely exploiting these locutions to efficiently and perspicuously describe relations and structures that they do believe in. Indeed, when challenged to explain what he means, a competent metereologist will show how all reference to centrifugal forces can be eliminated in favor of more longwinded claims about entities he does believe in. Another example is metaphor and figurative speech. When someone says he has butterflies in his stomach, there is an obvious sense in which this statement may be correct without being strictly and literally speaking true. But when pressed, this person will paraphrase his claim in a way that eliminates all reference to butterflies.

It is far from obvious, however, that a similar argument can be made about mathematical discourse. Other things equal, a two-status interpretation of mathematical language is not very appealing. For unlike the two examples above, we have no clear conception of any "higher" notion of correctness with which our ordinary notion of mathematical correctness can be contrasted. When we make a mathematical assertion, we *already* seem to be fully committed to its strict and literal truth. And when pressed, we're not inclined to qualify our assertion or to paraphrase it so as to eliminate mathematical vocabulary; indeed, in general we're not even *able* to give any such paraphrase.

Of course, this does not establish that a two-status interpretation of mathematical language is unacceptable.[4] For other things may *not* be equal. What I have argued is just that there is a general presumption in favor of one-status interpretations, and that facts about people's use of mathematical language provide no grounds for overriding this presumption. Thus, if a two-status interpretation of mathematical language is to be defended, ontological considerations will have to carry *all* the weight. In the next section, I will begin to develop an argument that ontological considerations cannot carry this weight.

3. *Analyzing the Concept of an Object*

So far, we've been discussing ontology in a somewhat uncritical manner. Let's now attempt to be a bit more careful. Ontology, we are told, attempts to determine whether there are such objects as events, arbitrary mereological sums, possible worlds, and numbers. But what can we say about the central concept of ontology, the concept of an object?

At first, it is hard to see that there is a question here at all. Surely, the concept of an object must be primitive and indefinable. As Frege observes, this concept is "too simple to admit of logical analysis," and "what is logically simple cannot have a proper definition."[5] Nor do we need any definition; for in practice, we all seem to know what an object is. We talk about objects, we distinguish objects from one another, we count objects, and we recognize separate glimpses as being of the same object. Underlying these capacities and practices, it seems, there must be some concept of an object that we all understand, at least implicitly. And when philosophers discuss ontological questions, we are just relying on this ordinary concept that everyone understands.

Although Frege is surely right that a "proper definition" of the concept of an object is out of the question, I think it is reasonable to ask for some further explication of this concept. We may, for instance, inquire what our grasp of it *consists in*. We would like to know what this concept allows us to do and what role it plays in our thought. In this section, I will argue that our ordinary talk about objects or things has two distinct sources that correspond to two distinct concepts of an object.

Imagine you ask a non-philosopher what an object is. Most likely, the answer will be something like, "Well, a thing, like a table or a chair or a rock or a planet." These objects are all bodies. So one sense of the word 'object' is *body*. Roughly speaking, a body is a solid spatiotemporal object with relatively well distinguished boundaries that moves as a unit.[6] Apples, for instance, are bodies, whereas pairs of apples, undetached apple-halves, and one-second apple-stages are not (because their spatial or temporal boundaries are too unnatural).

Obviously, the concept of a body is too narrow to be identified with the concept of an object that is used in ontology. Whatever events, arbitrary mereological sums, possible worlds, and numbers are, they are not bodies. I propose to analyze this trivial observation as follows. Let's say that a concept of an object is *perfectly general* if its generality is such that no analysis of it suffices to negatively settle ontological questions. In other words, for a perfectly general concept of an object and a coherent predicate *F*, it is never analytic that there are no *F*s. Since it is analytic that no bodies are numbers, the concept of a body lacks perfect generality.[7]

Contemporary ontological views with parsimonious leanings require a perfectly general concept of an object. For instance, when nominalists deny that there are numbers, they do not mean to deny that there are such *bodies* as numbers. That claim would be boring. What they mean to deny is that the numerals refer to *objects of any sort*. This claim is not meant to follow simply from an analysis of the concept of an object that is brought to bear. It is intended to be a substantive metaphysical claim.[8] But if a negative existential claim is to be substantive, we need a perfectly general concept of an object.

Perhaps the second root of our ordinary talk about objects is better suited for ontological theorizing. This root lies in the referential apparatus of language or of thought—for concreteness, I'll here focus on language[9]—and it corresponds to a second and more abstract sense of the words 'object' or 'thing'. Natural language contains a large formal apparatus for talking about individual things. There is a grammatical category of proper names, there are various devices for cross-reference, and there are rules that allow us to make and reason with generalizations of various sorts. One part of this formal apparatus is "the logic of identity." Terms that purport to denote objects, as opposed to quantifier phrases and non-nominal expressions, must be able to occur meaningfully in identity statements (at least within their own sort, if the language is many-sorted). Another part is "the logic of quantification," which consists of natural language equivalents of rules of inference such as Existential Generalization and Existential Instantiation. These devices, which are given a formal representation in first-order quantification theory, jointly give content to the logical concept of an object.[10]

For the most part, this concept surfaces linguistically only through the use of pronouns and quantifier expressions. In some contexts, however, the words 'object' or 'thing' are explicitly used to represent it. This is often the case with propositional attitude contexts.[11] Consider, for example, the following two sentences:

(1) Alice is thinking of last 4th of July, and Bob is thinking of the same thing.

(2) Alice is fascinated by the number B, and Bob is fascinated by the same thing.

Here the expression 'same thing' serves to facilitate cross-reference: It refers back to a day or to a number. We could easily construct similar sentences where the expression 'the same thing' refers to any other kind of entity.

The first person to develop a formal language in which the logical concept of an object can be represented was Frege. He did this in his *Begriffsschrift* of 1879, where he presented a formalization of second-order logic with polyadic relations and iterated quantification, essentially as we know it today. As we know, this represents a vast increase in expressive and deductive power over the earlier, essentially Aristotelian, logics.

However, to arrive at the logical concept of an object, Frege also had to break with the traditional *interpretation* of logic. Traditional Aristotelian logic was tied to an essentially Aristotelian metaphysics. The distinction between the logical categories of subject and predicate was tied to the ontological distinction between particulars and universals: Subject expressions were taken to refer to *particulars*, and predicate expressions were assumed to stand for *universals*. The concept of a particular was in turn closely connected to that of a *substance*. Along with his rejection of the formal logic derived from Aristotle, Frege rejected the ontological distinction between particulars and universals. In its stead, he proposed a semantic analysis based on the mathematical concepts of *function* and *argument*, to which correspond two new ontological categories, namely *concepts* and *objects*. He took concepts to be functions,[12] and objects to be anything that can serve as arguments of first-order functions.[13]

The logical concept of an object results from combining these two contributions of Frege's. Objects are thus characterized as the sort of entities that, firstly, serve as referents of singular terms of a first-order language, and secondly, compose its domain of quantification. This concept of an object is as general as the formal logic that gives content to it. And as Frege observes, this logic can be applied not only to what is physically and psychologically real or to what can be visualized, but to everything thinkable.[14] It follows that the logical concept of object is perfectly general. The logical concept of an object is therefore of great importance to ontology. Prior to Frege's explication of this concept, many of the ontological questions discussed in contemporary metaphysics couldn't even be properly formulated: It is much less plausible that there exist such *substances* as events, arbitrary mereological sums, and numbers than that the corresponding *objects* exist.[15]

However, the logical concept of an object is inadequate, by itself, to answer ontological questions. To see this, we need to introduce the concept of a sortal. I'll say that a concept *F* is a *sortal* if it satisfies the following two conditions. First, *F* provides criteria of identity for all objects falling under it. That is, whenever *a* and *b* are *F*s, *F* provides truth-conditions for the claim '$a = b$'. Second, it makes sense to ask how many *F*s there are. Note that I am not requiring that for *F* to be a sortal it provide some particular method or algorithm for answering these questions. All I require is that the questions be meaningful.

Two examples of sortals are the concepts of *body* and *natural number*. As we've seen, the concept of a body applies to solid spatiotemporal objects with relatively well distinguished boundaries that move as units. This allows us to make sense of questions about the individuation of bodies and their cardinality. Natural numbers are (arguably) individuated by what is known as *Hume's Principle*, which says that the number of *F*s is identical to the number of *G*s if and only if the *F*s and the *G*s can be one-to-one correlated.

But the logical concept of an object is not a sortal. This concept is purely formal, in the sense that its content derives solely from the structure of the referential apparatus of language or of thought. So there are no constraints, intrinsic to this concept, on how it can be applied. Imagine, for instance, you're looking at a table on which are placed four apples and nothing else. If I ask you *How many objects?* you will most likely interpret me as asking *How many bodies?* and answer *Four*. But assume I tell you I'm not interested in the number of bodies but in the number of objects in the sense of the logical concept of an object. Then you wouldn't know what to answer. For the logical concept gives you no clues as to how objects are to be individuated and re-identified. Depending on what concepts you bring to bear, you may regard the scene as consisting of two apple-pairs, eight apple-halves, an abundance of one-second apple-stages, and so on indefinitely. As far as the logical concept of an object is concerned, all of these are acceptable objects.

To ask *How many objects in the sense of the logical concept?* is a bit like asking *Which is the third?* without specifying any ordering. Clearly, this latter question makes no sense. To be meaningfully applied, the concept *the third* must be supplemented by some relation. For instance, a man or an apple cannot be said to be the third, only the third *tallest living person* or the third *largest apple on the table*.

In the same way, to make specific ontological claims, the logical concept of an object must be supplemented by some sortal. Assume I want to claim that there are or are not *F*s. Then a sortal is required to specify what it would take for there to be an *F*. Unless this is specified, no ontological claim will have been made. In some cases, *F* itself is a sortal; in other cases, *F* is canonically associated with some sortal, such as *criminal* is with the sortal *person*. But if no sortal is involved, no definite claim has been made. For instance, as we've seen above, it makes no sense to claim that there are three objects in the sense of the logical concept of an object.

To sum up, I have argued that our ordinary talk about objects has two distinct sources. One source corresponds to the concept of a body. This concept is a sortal. But it lacks the perfect generality that is needed for negative ontological claims to be substantive. The other source corresponds to the logical concept of an object. This concept inherits perfect generality from the formal logic that gives content to it. But this formal character also prevents it from being a sortal.

Call a concept of an object that is *both* perfectly general *and* a sortal an *absolute* concept of an object. My argument in this section makes plausible, but doesn't yet establish, that we possess no absolute concept of an object. Before I attempt to establish this claim, in Section 5, I would like to have a look at its philosophical payoff. In the next section, I will therefore present the central argument of this paper, which depends on the claim that we possess no absolute concept of an object.

4. *Towards a One-Status Approach*

What I just called "the central argument" aims to show that the fundamental tension is spurious and that it is possible to defend one-status interpretations of the relevant discourses. I will first sketch a general strategy for developing one-status approaches, and then apply the strategy to some simple examples. In Section 6, I will show how the strategy can be extended to some harder cases.

The strategy proceeds as follows. If our only perfectly general concept of an object is the formal one associated with quantificational logic, then, in order to talk about specific objects or to assert the existence or non-existence of some specific class of objects, we need to supplement this formal concept with some sortal. This sortal will be specific to the subject matter in question, and it will tell us what existence amounts to in this context. This yields an extremely flexible notion of objecthood. In particular, with this notion of objecthood, the existence of *F*s may amount to nothing more than what is expressed by the Correctness-conditions that the two-status theorists assign to the relevant existential sentences. This opens up the possibility that the apparent divergence of standards that gives rise to the fundamental tension may be spurious.

To illustrate this strategy, I'll now consider two examples. These examples are quite simple, in the following precise sense: Although they are concerned with discourses that are committed to "problematic" objects, these discourses have Correctness-conditions that can be expressed without commitment to the "problematic" objects in question. I will therefore say that we have a *reductive account* of these Correctness-conditions.

The first example assumes that we understand quantification over lines as well as certain predicates $f_1, f_2, \ldots, f_n$ that hold of lines and that are congruences with respect to parallelity.[16] Then we introduce the concept of a direction by laying down the following stipulations.

(D1) '$d(l) = d(l')$' is true iff $l \,//\, l'$

(D2) '$F_i(d(l))$' is true iff $f_i(l)$

(D3) '$\exists d.\varphi(d)$' is true iff there is a line l such that '$\varphi(d(l))$' is true.

These stipulations describe a new language in which we can refer to directions, apply certain predicates $F_1, F_2, \ldots, F_n$ to directions, and quantify over directions. Moreover, since the stipulations allow us to express the truth-conditions of any sentence of the new language of directions in terms of the old language of lines, there can be no doubt about the logical coherence of the new language.

However, since directions are abstract, the nominalists insist that we should not accept these entities. So they deny that there are objects of the sort to which D-terms are intended to refer. If the nominalists are right about this, existential D-sentences will strictly and literally speaking be false—at least if their syntactic and semantic forms are what they appear to be. However, despite their claim that D-terms fail to refer, the nominalists grant that the stipulations which introduce these terms are logically coherent and that they set up sharp and objective conditions for the Correctness of D-sentences. In short, the nominalists pursue a standard two-status approach to D-discourse.

I contend that this two-status approach is untenable because we cannot make sense of the nominalist's rejection of directions in a way that makes this claim both substantive and true. To see this, formalize the claim as $\neg\exists x.\mathrm{D}x$. If the concept of an object associated with the existential quantifier is that of a body, the claim will certainly be true. But thus construed, it will be trivial and completely uninteresting. No platonist has ever claimed that directions are bodies. In order to make a substantive claim, the nominalists will have to employ a perfectly general concept of an object. But if I am right, our only perfectly general concept of an object is the formal one provided by first-order logic. What the claim says will then be determined by the sortal corresponding to the predicate 'D'. But according to the conditions associated with this sortal, it is the opposite claim, $\exists x.\mathrm{D}x$, that is true.

Of course, the nominalists may reject my claim that our only perfectly general concept of object is the logical one. I will attempt to defend this claim in the next section. Apart from that, the nominalist has only two ways of responding to my argument. One option is to deny that D is a genuine sortal. But this does not seem promising. Since the nominalists have granted that the Correctness-conditions associated with D are coherent, sharp, and objective, they cannot deny that D is a genuine sortal.

A second option is to deny that the expression '∃d' that occurs on the left-hand side of (D3) really is an existential quantifier. This option seems somewhat more promising. For by embracing a so-called *reductionist reading* of the stipulations (D1)-(D3), it can be argued that their left-hand sides have been assigned meanings only as unarticulated wholes and not in a way that reflects their apparent syntactic structures. However, since the stipulations warrant application to '∃d' of the logical rules that give content to the logical concept of an object, it cannot be denied that this expression *functions logically* as an existential quantifier. And this is all we need to establish that it is a quantifier corresponding to the logical concept of an object. The only worry is that it may be a substitutional quantifier rather than an objectual one. But in this case, there isn't much difference between objectual and substitutional quantification. Since every direction is denoted by a D-term (with one or more free variables ranging over lines), both sorts of quantification have the same range. Moreover, even when the quantifiers are interpreted substitutionally, a formula is existentially committed to the semantic values of appropriate instances of the variables in question. But on the view in question, the existence of directions consists in nothing more than does that of the semantic values of the corresponding D-terms.

The second simple example of my general strategy begins with a language with plural quantification and certain plural predicates $g_i(xx)$.[17] In this language, we then lay down the following stipulations, which associate with the objects *xx* a unique mereological sum s(*xx*) and introduce certain new predicates $G_i(x)$ that hold of mereological sums:

(M1) '$s(xx) = s(yy)$' is true iff $\forall u(u$ is one of $xx \leftrightarrow u$ is one of $yy)$

(M2) '$G_i(s(xx))$' is true iff $g_i(xx)$

(M3) '$\exists s.\varphi(s)$' is true iff there are objects *xx* such that '$\varphi(s(xx))$' is true

As in the case of directions, these stipulations allow us to express the truth-conditions of sentences of the new language in terms of sentences of the old. And since the appropriate existential sentences of the new language come out true, I claim that we cannot make sense of the mereological fictionalist's claim that there are no mereological sums in a way that makes this claim both substantive and true.

In both these examples we begin with a discourse that isn't committed to *F*s and lay down truth-conditions for sentences that appear to be thus committed. Since some of these sentences come out true, we claim that there exist *F*s. But where do these objects come from? It looks like we pulling rabbits out of a hat. In particular, how can a sentence that is committed to *F*s be analytically equivalent to one that isn't?

The answer lies in the great flexibility of the logical concept of object. This concept allows the content of existential claims to vary greatly from one field to another. Since there is no absolute concept of an object, the logical concept of object must be supplemented with local sortals, which specify what existence amounts to in this field. But these sortals vary greatly: some impose very stringent conditions; others, only very lax ones. I will call objects corresponding to the former kind of sortals *thick*, and objects corresponding to the latter *thin*. The concept of a physical object, for instance, is tied up with physics, and its instances will therefore, as a necessary but not sufficient condition, have to satisfy the laws of physics and be suitably linked to observation. So physical objects are quite thick. In contrast, mathematical objects are very thin. They do not have to satisfy the laws of physics or be in principle observable.[18]

5. *Is There an Absolute Concept of an Object?*

But aren't these entities simply too thin to deserve to be called objects? They don't seem to be what we ordinarily mean by the word 'object'. Objects are supposed to be items that the world contains. And what the world contains cannot be investigated by methods such as those I have used to defend the existence of directions and mereological sums.

I believe this worry is both strongly felt and widely shared. However, by assuming that an object is simply an item that the world contains, this worry presupposes what I have called *an absolute concept of objecthood*, that is, a concept of object that is both perfectly general and a sortal. Do we possess such a concept? And if we do, what does our grasp of it consist in?

An absolute concept of an object is most plausible when based on the metaphysical view that all of reality is uniquely "carved up" into "natural units"[19] in a way that is independent of our language and our thought. But this view is extremely abstract and has so far only been described in highly metaphorical terms. If this view is to bear any philosophical weight, the complimentary metaphors of "carving out" and "natural unit" need to be spelled out, and we need to assure ourselves that we have a satisfactory grasp of these notions. True, we know what it means to carve out a statue from a lump of clay, and we know that the statue, once carved out, is a natural unit. But the metaphysical view in question puts far greater demands on the concepts of "carving out" and "natural unit" than does this example: For this view to work, these concepts will have to be perfectly general; otherwise, they will restrict the generality of the resulting concept of an object.

Our paradigmatic example of natural units are bodies. I take it to be uncontroversial that bodies are natural units in some sense that distinguishes them from most other kinds of object. Consider, for instance, the table on which are placed four apples and nothing else. It cannot be denied that ordinary apples are natural in a way that apple-pairs, apple-halves, and one-second apple-stages are not. Perhaps this naturalness can be used to give content to an absolute concept of an object. When we observe that bodies are natural units, perhaps we are acquainted with some notion of naturalness that is general enough to give content to the picture of reality as consisting of a determinate range of natural units.

But this proposal fails. The problem is that the sense in which bodies are natural units is specific to bodies and thus insufficiently general to give content to the metaphysical picture in question. Consider again the concept of a body: a three-dimensional material object with relatively well-distinguished boundaries that moves as a unit. This concept *spells out* the way in which bodies are natural units: Their naturalness is characterized by a concern for spatiotemporal boundaries and units of independent motion. But this means that the naturalness exemplified by bodies is too narrow to give content to the metaphysical picture.

Perhaps physics is better suited to give content to an absolute notion of naturalness.[20] For according to physics, things that aren't themselves natural units—such as clouds, oceans, and forests—are nevertheless composed of elementary particles that are. In response to this modified proposal, I confess I don't know how to analyze the naturalness that elementary particles possess. But I am confident that an adequate analysis will have to make reference to spacetime and causal efficacy, and thus again be too narrow to be universally valid. More generally, I conjecture that other uncontroversial cases of natural units will suffer the same fate: They too will turn out to be natural only in some way that is specific to the kind of object in question.

So it seems that the adherent of an absolute concept will have to change his strategy. Perhaps he should grant that we're unable to give any informative account of our grasp of this concept and argue instead that we have general philosophical reasons to postulate such a concept. Although I find this strategy suspicious, I will now consider two attempts to develop it.

The first attempt is based on the observation that it *strongly seems to us* as if we possess an absolute concept of an object. We have a strong intuition that it makes sense to talk about the unique range of objects that reality contains. Unless we have some specific reason to reject this intuition, we should take it at face value and thus assume that we possess an absolute concept.

I acknowledge the intuition on which this argument is based and feel the force of it myself. However, I believe this intuition is an illusion that my view can account for. Kant famously compared his so-called "transcendental illusions" to optical illusions in their inevitability and their independence of theoretical knowledge.[21] Even if I *know* that the oar in the water is straight, it still *seems to me* that it is bent, and no amount of theoretical reflection can shake this impression. The idea that we possess an absolute concept of object seems to me similar: Even when we have convinced ourselves that we possess no absolute concept, we still find ourselves slipping back into the old mode of thought. However, by attending to the two distinct sources of our ordinary talk about objects, we are able to explain why we are subject to this illusion. For the concept of a body and the logical concept of an object each possess one of the defining features of the absolute concept. When we use the word ‘object’ in the sense of *a body*, we are dealing with a sortal that applies to natural units. And when we use the word ‘object’ in the sense of the logical concept, we are employing a concept that is perfectly general. It is therefore natural to assume that these two features can be combined into a single concept of an object, which would then be an absolute concept. However, as we’ve seen, careful analysis favors the opposite view that each of these features is possible only at the expense of the other.

The second attempt argues that unless we postulate an absolute concept of an object, disastrous philosophical consequences will ensue.[22] For instance, without an absolute concept of naturalness, we may be unable to do justice to the datum that bodies are natural units. If the naturalness that bodies possess isn’t absolute but specific to the concept of a body, then it seems somehow subjective or ideal.[23] More generally, one may worry that, by rejecting the absolute concept of an object, we’ll be forced to accept Hilary Putnam’s doctrine of “conceptual relativity,” according to which all facts are relative to our conceptual scheme.[24] For instance, one may worry that the concept *electron* will no longer have a privileged role in the description of the universe.

Fortunately, my view isn’t committed to any such consequences. The undesirable conclusion—that bodies are no more natural than are body-pairs, body-halves, and one-second body-stages, and that there is no privileged way of “slicing up reality”—would follow only if we were to grant that all coherent sortals are on a par. But we can reject the idea of an absolute concept of an object without embracing any such conceptual egalitarianism. For even without an absolute concept, it will be possible to maintain that some coherent sortals are uninteresting or simply bad. In fact, we may even hold that to each domain of inquiry there correspond a unique system of fundamental sortals. For instance, there may be a unique best way of describing the basic building blocks of the physical universe. If so, the doctrine of conceptual relativity will be banished to non-fundamental ways of describing reality. And thus understood, the doctrine should be neither surprising nor particularly worrysome.

I conclude that we can, without philosophical danger, deny that we possess an absolute concept of an object. And when we do so, we have no choice but to accept thin objects. Although such objects may at first seem counter-intuitive, the fact that they allow for one-status approaches to the fundamental tension shows that they should be welcomed rather than perceived as a threat.

6. *What Mathematical Objects Could Be*

In this final section, I will sketch a general view of mathematical objects according to which such objects are very thin. In slogan form, the view is that the existence of a class of pure mathematical objects amounts to nothing more than the *logical coherence* of the theory that describes them. All mathematical objects do is to "span" concepts; that is, to guarantee that the relevant concepts are coherent.

This view can be regarded as a reversal of Tarski's well-known characterization of logical consequence in terms of the existence of abstract mathematical models. Looking at logical coherence instead of logical consequence, Tarski's claim is that a theory T is logically coherent just in case it is satisfiable; that is, just in case T has a model. Whereas Tarski regards the right-hand side of this biconditional as an explication of the left-hand side, I propose to reverse the explanatory direction and use logical coherence to explicate mathematical existence.

This proposal gives rise to a number of questions, of which the following three seem to me the most important. First, what is this notion of logical coherence? Second, is this really what mathematicians mean when they assert that various kinds of mathematical objects exist? And third, do mathematical objects, thus understood, still give rise to nominalistic worries? Although each of these questions deserves a paper-length response by itself, all I can do here is briefly describe the kinds of answers I find promising. The remarks I am about to give should therefore be read more as an advertisement for a research program than a defense of a fully worked out view.

Loosely speaking, a theory T is said to be logically coherent if we have a conception of a structure in which T would be true; or, more precisely, if we have a conception of a structure in which every sentence of the language of T is assigned a unique truth-value, and in which the theorems of T are all assigned the value *true*. For theories formulated in the language of classical first-order logic, this notion of logical coherence will be coextensive with that of syntactic consistency. But for stronger logics without a complete proof procedure, such as full second-order logic or first-order logic augmented with certain cardinality quantifiers, this equivalence fails. Such theories can be consistent and still fail to be logically coherent.[25]

Obviously, this characterization of logical coherence is rather schematic. But I doubt that this notion can be explicitly defined in more basic terms. So the question how this notion is to be characterized is a hard one that will have to be left open for now. As will become clear below, I take this to be the main question confronting any viable form of mathematical platonism.

Despite the difficulties involved in characterizing the notion of logical coherence, I submit that this notion, or something close to it, is assumed by almost all parties to the dispute about mathematical objects. In particular, all two-status approaches I am aware of are committed to mathematical theories' being coherent in this sense. For although they deny that mathematical theorems with abstract commitments are True, they grant that mathematical discourse is associated with strict and coherent standards of Correctness. Again, the views defended by Field and Yablo provide nice examples.

As we've seen, Field's account of mathematics has two steps. First, he attempts to develop nominalistic versions of various scientific theories of the world. In these theories, all quantifiers range over physical objects. Second, Field attempts to show that platonistic mathematics is conservative over these nominalistic theories. Both these steps make strong assumptions about the coherence of mathematical theories. The first step provides physical models in which various mathematical theories can be interpreted. For instance, the nominalistic theory of space in developed in Field (1980) is strong enough to interpret second-order real analysis. The second steps aims to show that a platonistic theory P is conservative over nominalistic science. But to show this is to show that P + N is coherent, where N is a nominalistic theory.[26] *A fortiori*, P itself will be coherent.

Yablo never spells out in precise terms the features that make platonistic mathematics pretense-worthy. But a minimal requirement is that the theory in question be coherent. So Yablo too grants enough for thin mathematical objects to exist.

Our second question was whether the analysis of mathematical existence I've proposed is faithful to what mathematicians themselves mean when they assert that mathematical objects exist. I will approach this question by way of a somewhat narrower one. In mathematics we often assure ourselves that a mathematical theory is coherent by finding a model of it. For instance, the theory of complex numbers is shown to be coherent by interpreting it in terms of pairs of real numbers, and the adjunction to number theory of certain "ideal numbers" is shown to be coherent by interpreting the resulting theory in terms of a simple set theoretic construction on polynominals over the integers. Don't these examples show that it is logical coherence that is explained in terms of mathematical existence rather than the other way round, as I have suggested?

I don't think so. In these proofs, one mathematical theory is shown to be coherent by interpreting it in some mathematical structure already assumed to exist. This process of interpretation cannot go on forever but must terminate in certain mathematical structures whose existence is taken as basic. On my neo-Dedekindian view, the existence of these basic structures should be cashed out in terms of their logical coherence. The model-theoretic proofs of coherence that these basic structures allow for can then be understood as relative proofs of coherence. This allows us to explain the success and wide-spread use of model-theoretic techniques without granting that the existence of models is more fundamental than the notion of logical coherence.

Turning now to the general question whether the view I've proposed of mathematical existence is faithful to mathematical practice, I claim that this view was held by a number of important nineteenth century and early twentieth century mathematicians, such as Georg

Cantor, Richard Dedekind, Henri Poincaré, and (the young) David Hilbert. I provide some textual evidence for this claim in an Appendix. If the view is less visible today, I think this is explained by the emergence of axiomatic set theory and its acceptance by ordinary mathematicians. Standard ZFC set theory is strong enough to interpret nearly all existing mathematical theories, and this allows us to give relative proofs of the coherence of nearly all mathematical theories. So for the most part, it is no longer necessary to investigate the coherence of mathematical theories by direct means. But the view that coherence suffices for mathematical existence is still with us, although it may be less visible today than it used to be. For when we convince ourselves of the coherence of structures that cannot be interpreted in set theory, we tend to expand the mathematical universe accordingly, for instance by accepting suitable large cardinals.

Finally, there is the question whether the view of mathematical objects I've defended remains subject to nominalistic worries. In Section 1, I described the three most wide-spread worries about abstract objects. I will now argue that these worries presuppose a view of mathematical objects as quite thick. My claim will be that mathematical objects are objectionable only insofar as they are assimilated to concrete objects. Of course, this does not mean that hard philosophical questions won't remain even when mathematical objects are regarded as thin. But I will argue that the questions that remain are no harder (and in fact a bit easier) for the thin platonist than for the two-status nominalist.

First, there was the worry that abstract objects will be epistemically inaccessible. As we saw, Field claims that "according to the platonistic picture, the truth values of our mathematical assertions depend on facts involving platonic entities that reside in a realm outside of space-time." From this he concludes that, in order to account for mathematical knowledge, the platonist has to "postulate some *aphysical connection*, some *mysterious mental grasping*, between ourselves and the elements of this platonic realm" (Field (1989), p. 68). But in fact, the explanatory burden that confronts the thin platonist will be no larger than that confronting Field. The thin platonist has to explain how it is possible to know facts about logical coherence. But this task is one that Field too faces. The impression that the platonist's task is so much harder is due to Field's tacit reliance on a thick notion of an object, which assimilates mathematical objects to concrete ones.

Next, there was the metaphysical worry that pure mathematical objects appear to have no properties other than structural ones. In particular, we saw that Field objected to mathematical platonism on the ground that there appear to be no definite answers to questions whether an object from one mathematical structure is identical with one from another. But on the view I've defended, the lack of answers to such questions comes as no surprise. Such questions could be expected to have context-independent answers only if there were an absolute concept of an object. But I have argued that there is no such concept, and that talk about objects and their identity is always relative to sortals that fix their identity conditions. So this relativity to sortals isn't peculiar to pure mathematical objects but a general feature of our talk about objects.

Finally, there was the worry that postulating an enormous domain of mathematical objects will be metaphysically extravagant and hence constitute poor ontological economy. We saw this worry at work when Field at one point compares mathematical objects to "little green people living inside of electrons and that are in principle undiscoverable by human beings" and then proceeds to point out that "it seems … undue epistemological caution to maintain agnosticism rather than flat out disbelief about such an idle hypothesis" (Field (1989), p. 45).[27] Of course, no one in his right mind would believe in undiscoverable green people inside of electrons. But what does this tell us about mathematical objects? Unless it is granted that mathematical objects are essentially like concrete objects, it tells us *absolutely nothing*. But in fact, it is precisely this comparison that the thin platonist rejects. He holds that mathematical objects are very different from concrete objects, and that this difference consists in part in the fact that logical coherence suffices for the existence of the former but not for that of the latter.

It emerges that two-status theorists have misdiagnosed the philosophical problem that platonistic mathematics represents. Most two-status theorists find it unproblematic that platonistic mathematics should possess Correctness-conditions that determine, for every mathematical statement, whether or not it is Correct. They also find it unproblematic that we should have an adequate grasp of these conditions. What they find problematic is that mathematical statements should be True and not merely Correct. For they think that, in order for such statements to be True, reality must contain certain platonic objects whose existence is not guaranteed by the mere fact that these statement are Correct and that the theory to which they belong is logically coherent. On their view, it is this *additional* requirement that makes platonistic mathematics problematic.

On my view these claims about what is problematic should be reversed. By denying that we possess an absolute concept of an object, and by emphasizing the great flexibility of our only perfectly general concept of an object, I have argued that the Truth of a mathematical statement requires nothing more than does its Correctness, as analyzed by the two-status theorist. I therefore deny that the Truth of mathematics represents a problem over and above that of its Correctness. On the other hand, I admit that even the thin mathematical objects I've defended give rise to some hard philosophical questions. For metaphysical and epistemological questions remain about the notion of mathematical concepts' being coherent. How is this notion of coherence to be analyzed? Does it provide a sufficient basis for mathematical objects to exist in some substantive way, or is this merely existence in some highly attenuated sense? How objective are matters of coherence? And how do we come to know about these matters?

No doubt, these are hard questions. But I am convinced they are more tractable than the corresponding questions about thick mathematical objects. On the view I've sketched, it is no longer a complete mystery how mathematical platonism can be true and how mathematical objects can be accessible to human knowledge.[28]

Appendix

From Cantor's *Foundations of a General Theory of Manifolds*:

> Mathematics is in its development entirely free and is only bound in the self-evident respect that its concepts must both be consistent with each other and also stand in exact relationships, ordered by definitions, to those concepts which have previously been introduced and are already at hand and established.
>
> In particular, in the introduction of new numbers it is only obligated to give definitions of them which will bestow such a determinacy and, in certain circumstances, such a relationship to the older numbers that they can in any given instance be precisely distinguished. As soon as a number satisfies all these conditions it can and must be regarded in mathematics as existent and real. […] But every superfluous constraint on the urge to mathematical investigation seems to me to bring with it a much greater danger, all the more serious because in fact absolutely no justification for such constraints can be advanced from the essence of the science – for the *essence* of *mathematics* lies precisely in its *freedom*. (Cantor (1883/1996), p. 896)

From Poincaré's "Mathematics and Logic, I":

> Mathematics is independent of the existence of material objects; in mathematics the word 'exist' can have only one meaning; it mean free from contradiction. (Poincaré (1905/1996), p. 1026)

From Poincaré's "Mathematics and Logic, III":

> What does the word *exist* mean in mathematics? It means, I said, to be free from contradiction. This M. Couturat contests. 'Logical existence', he says, 'is quite another thing from the absence of contradiction. It consists in the fact that a class is not empty.' And doubtless to affirm that the class *a* is not null is, by definition, to affirm that *a*'s exist. But one of the two affirmations is as denuded of meaning as the other, if they do not both signify, either that one may see or touch *a*'s, which is the meaning physicists or naturalists give them, or that one may conceive an *a* without being drawn into contradictions, which is the meaning given them by logicians and mathematicians. (Poincaré (1906/1996), p. 1055)

From Hilbert's "Mathematical Problems":

> To show the significance of the problem from another point of view, I add the following observation: If contradictory attributes be assigned to a concept, I say, that *mathematically the concept does not exist*. … But if it can be proved that the attributes assigned to the concept can never lead to a contradiction by the application of a finite number of logical inferences, I say that the mathematical existence of the concept … is thereby proved. (Hilbert (1900/1996), p. 1105)

Notes

[1] For different kinds of fictionalism about mathematics, see Balaguer (1996); Field (1980) and (1989); Melia (1995) and (2000); and Yablo (2000) and (MS). Fictionalism about mereological sums is defended in Dorr and Rosen (forthcoming), and fictionalism about modality, in Rosen (1990).

[2] See Field (1980) and (1989).

[3] See Yablo (2000) and (MS).

[4] For an attempt to do that, see Stanley (2002).

[5] See, respectively, Frege (1891/1952), p. 32 and Frege (1892/1952), p. 43.

[6] Or would so move if it did. Henceforth, I will suppress this qualification.

[7] The notion of analyticity relied on here is very weak. It is comparable to that involved in the claim that it is analytic that no bachelor is married, which even Quine has retrospectively recognized as true. See e.g. Quine (1991), p. 270.

[8] See, for example, Field (1993) and Yablo (2000).

[9] I intend to keep my discussion noncommittal as to whether this concept is primarily associated with language or with thought.

[10] The role that proper names, pronouns, and other devices of cross-reference play in natural language are captured by singular term and variables. And some of the most important quantificational devices of natural language can be represented by means of the existential and the universal quantifiers. However, since natural language contains a plethora of referential and quantificational devices, whereas first-order logic contains only a small number of primitive logical devices, it is not obvious that first-order logic is sufficient to represent the concept of an object associated with natural language. Nevertheless, I believe that first-order logic does suffice.

[11] I believe this is because propositional attitudes constructions, unlike most other predicates, take arguments from any ontological category. For instance, it is possible to think of everything from rocks and people to numbers, places, and intervals of time.

[12] Frege first took concepts to be functions from objects to so-called judgeable contents; later, with the introduction of the sense/reference distinction, he took their range to consist of the truth-values. See, respectively, Frege (1879/1967), Section 9 and Frege (1891/1952).

[13] These two contributions of Frege's are logically independent. Clearly, it is possible to stipulate that the universe of discourse of Frege's logic is to consist of substances only. Conversely, it is possible to combine Aristotelian logic with a more extravagant ontology than an Aristotelian one of substances. For instance, in the judgment "7 is prime," the numeral '7' could be said to denote the object 7. There is some evidence that Kant entertained a view of this form. See Allison (1983), pp. 135-6 and Parsons (1982). However, to get the full effect of Frege's liberation of logic from traditional ontology, we need a logic with greater expressive and deductive resources than Aristotle's. To see this, consider the numbers. What matters about the numbers are not their intrinsic properties (if any) but their relations to other numbers. But to express these relations, we need to go beyond Aristotelian logic.

[14] For this characterization of logical notions, see Frege (1884/1953), Section 14.

[15] Dummett makes a similar observation in (1981), pp. 471 and 478.

[16] That is, when l and l' are parallel, f_i holds of l if and only if it holds of l'.

[17] A language of this sort is assumed in Dorr and Rosen (forthcoming), who use it to defend mereological fictionalism. For an explanation of plural quantification, see Boolos (1984).

[18] In the final section, I will suggest that the only conditions mathematical object have to satisfy are certain minimal logico-mathematical ones.

[19] I here ignore the possibly of so-called "atomless gunk"; that is, stuff with no atomic parts. (For a precise characterization of atomless gunk, see Lewis (1991), pp. 20-21.)

[20] A similar suggestion is made at Lewis (1984), pp. 66-67, where Lewis appeals to physics to defend a notion of naturalness applicable to *classes* of objects.

[21] See Kant's *Critique of Pure Reason*, A298/B354-5.

[22] This is the main strategy that Lewis (1984) employs in response to Putnam's so-called "model theoretic argument."

[23] Michael Ayers, for instance, believes that if some concept is involved in our apprehension of bodies, some such ideality will inevitably follow. See e.g. Ayers (1997), where he contrasts the "natural or real" with the "conceptual or ideal" (p. 395).

[24] See Putnam (1990), essay 6, but also Putnam (1987) and (1988).

[25] For instance, let $E_{<\omega}$ be a sentence that says that there are less than T objects in the domain, and $E_{>n}$ one that says there are more than n. Then the theory whose set of axioms is $\{E_{<\omega}\} \cup \{E_{>n} \mid n \in \boldsymbol{N}\}$ will be syntactically consistent but logically incoherent.

[26] We need to require that N be agnostic about abstract entities. This can be ensured by restricting all of its quantifiers to concrete entities.

[27] The passage I just quoted is not unusual in Field. His polemic against platonism contains lots of passages where mathematical objects are assimilated to concrete objects. For instance, Field dismisses various *a priori* arguments for the existence of mathematical objects by comparing such arguments to *a priori* arguments for the existence of God or of Santa Claus. See Field (1989), p. 80 and p. 5, respectively. In fact, even a superficial inspection of our "theory of God" (if we follow Field in treating religious language as a scientific theory) reveals that the concept of a God is such that any object falling under it must be "maximally thick." Field's comparison is therefore a very bad one.

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